Unit – 1

STABILITY OF SLOPES

Introduction

An earth slope is an unsupported inclined surface of a soil mass. Earth slopes are formed for Railway formations, highway embankments, earth dams, levees etc.

The cost of earthwork would be minimum if the slopes are made steepest. However, steep slopes are not stable. Therefore, steepest slopes which are stable and safe should be provided.

The failure of a soil mass occurs along a plane or a curved surface when a large mass of soil slides with respect to the remaining mass.

The factors leading to the failure of slope may be classified into two categories.

(i) The factors which cause an increase in shear stress
  (a) Stress may increase due to weight of water causing saturation of soils
  (b) Surcharge loads
  (c) Seepage pressure
  (d) Steepening of slopes either by excavation or by natural erosion

(ii) The factors which cause a decrease in shear strength of the soil
  (a) Due to increase in water content
  (b) Increase in pore water pressure
  (c) Weathering or any other cause

- Most important forces which cause instability are the force of gravity and force of seepage (Instability : tending to move only, failure: actual movement of soil mass)
- Actuating forces induce shearing stress. Unless the shearing resistance on every possible failure surface within the soil mass is larger than the shearing stress, failure will occur in the force of movement of soil along a slip surface. (Actuating forces: forces causing instability)

BASIS OF ANALYSIS

The most common methods of slope analysis are based on limiting equilibrium in which it is assumed that the soil is on the verge of failure.

The methods of limiting equilibrium are statically indeterminate.

As stress - strain relationships along the assumed surface are not known, it is necessary to make assumptions so that the system becomes statically determinate.

- Shear strength = Max. Value of shear stress that can be mobilised.

GENERAL ASSUMPTIONS

1. The stress – system is assumed to be two dimensional. The stresses in the third dimension (perpendicular to the section of soil mass) are taken as zero.
2. Coulomb equation for shear strength is applicable and shears strength parameters c and Φ are known.
3. Seepage conditions and water levels are known, and the corresponding pore water pressures can be estimated.
4. The conditions of plastic failure are assumed to be satisfied along the critical surface. In other words, the shearing strains at all points of the critical surface are large enough to mobilise all the available shearing strength.
5. Depending upon the method of analysis, some additional assumptions are made regarding the magnitude and distribution of forces along various planes.

In the analysis, the resultant of all the actuating forces trying to cause the failure is determined. An estimate is also made of the available shear strength. The factor of safety of the slope is determined from the available forces and the actuating forces.

Factors of safety

The task of analyzing slope stability is to determine the factor of safety.

Three different definitions of factor of safety are used.

1. Factor of safety with respect to shear strength

The factor of safety is defined as the ratio of average shear strength of the soil to the average shear stress developed along the potential failure surface.

$$F_S = \frac{\tau_f}{\tau_m}$$

F_s = Factor of safety w.r.t strength

τ_f = avg. shear strength of the soil

τ_d = avg. shear stress developed along the potential failure surface.

The shear strength of a soil emits of two components, cohesion and friction and may be written as

$$\tau_f = c' + \sigma' \tan \Phi'$$

Where c′ = cohesion, Φ′ = angle of friction

σ′ = normal stress on the potential failure surface

In a similar manner, τ_d may be smaller as

$$\tau_d = c_d^{'} + \sigma' \tan \Phi_d^{'}$$

$c_d^{'}$ and $\Phi_d^{'}$ are cohesion and angle of function that develop along the potential failure surface.

$$\therefore F_s = \frac{c' + \sigma' \tan \Phi'}{c_d^{'} + \sigma' \tan \Phi_d^{'}}$$

(or) $F_S(c_d^{'} + \sigma' \tan \Phi_d^{'}) = c' + \sigma' \tan \Phi'$

(or) $c_d^{'} + \sigma' \tan \Phi_d^{'} = \frac{C'}{F_S} + \frac{\sigma'}{F_S} \tan \Phi'$

$$\therefore c_d^{'} = \frac{C'}{F_S} \quad \& \quad \tan \Phi_d^{'} = \frac{\tan \Phi'}{F_S}$$

These two expressions indicate that the factor of safety with respect to cohesion intercept and that with respect to angle of shearing resistance are equal to the factor of safety with respect to shear strength.

2. Factor of safety with respect to cohesion

Factor of safety with respect to cohesion (F_s) is the ratio of the available cohesion intercept (c) and mobilised cohesion intercept.

$$F_c = \frac{C'}{c_d^{'}}$$

c' = cohesion intercept and $c_d^{'}$ = mobilised cohesion intercept

3. Factor of safety with respect to friction

Factor of safety with respect to friction (F_Φ) is the ratio of the available frictional strength to the mobilised frictional strength.

$$F_\Phi = \frac{\sigma' \tan \Phi'}{\sigma' \tan \Phi_d^{'}} = \frac{\tan \Phi'}{\tan \Phi_d^{'}}$$

$$\Phi' = angle\ of\ shearing\ resistance$$

$$\Phi_d^{'} = mobilised\ shearing\ resistance$$

For small angles,

$$F_\Phi = \frac{\Phi^{'}}{\Phi_d^{'}}$$

In the analysis of stability of slopes, the three factor of safety are equal i.e.,

$$F_s = F_c = F_\Phi$$

When greater reliance is placed on Φ and c, FS w.r.t. cohesion is taken greater than that w.r.t. friction. In such case F_Φ is taken ad unity i.e., $\Phi = \Phi_d'$

- When $F_s = 1$, the slope is in as state of impending failure.
- Fs = 1.5 is generally acceptable for design of a stable slope.

Note:

When $F'_c = F'_\Phi$ it gives FOS with respect to strength. Or if

$$\frac{C'}{c_d'} = \frac{\tan \Phi'}{\tan \Phi_d'}$$

Then we can written

$$F_s = F_c = F_\Phi$$

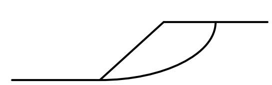
TYPES OF SLOPE FAILURES

A slope may have any one of the following types of failures.

1. **Rotational failure**
   This type failure occurs by rotation along a slip surface by downward and outward movement of the soil mass.
   The slip surface is generally circular for homogeneous soil and non – circular in the case of non – homogeneous conditions.

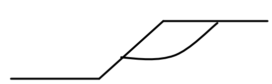
   Rotational slips are further divided into 3 types

   (a) Toe failure:
   - The failure occurs along the surface that passes through the toe.
   - Toe failures are most common.



Toe failure

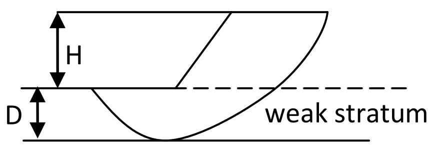
   (b) Slope failure:
   - The failure occurs along a surface that intersects the slope above the toe.
   - Slope failure occurs when a weak plane exists above the toe.



Slope failure

(c) Base failure

- ➢ The failure occurs along a surface that passes below the toe.
- ➢ Base failure occurs when a weak stratum lies beneath the toe. If a strong stratum exists below the toe, the slip surface of the base failure is tangential to that stratum.



Base failure

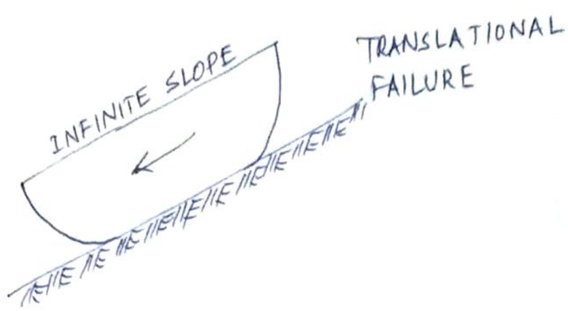
D_f = Depth factor = $\frac{(H+D)}{H}$

For toe failures, $D_f = 1$

For Base failure. $D_f > 1$

2. Translational failure

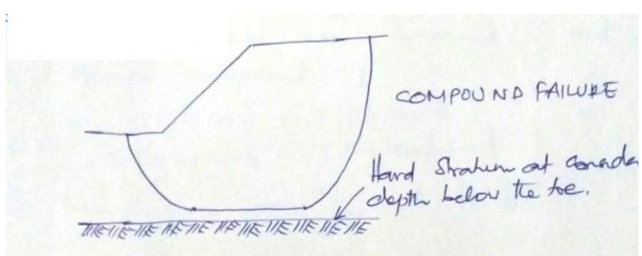
- ➢ Translational failure occurs in an infinite slope along a long failure surface parallel to the slope.
- ➢ The shape of the failure surface is influenced by the presence of any hard stratum at a shallow depth below the slope surface.
- ➢ Translational failure may also occur along slopes of layered materials.



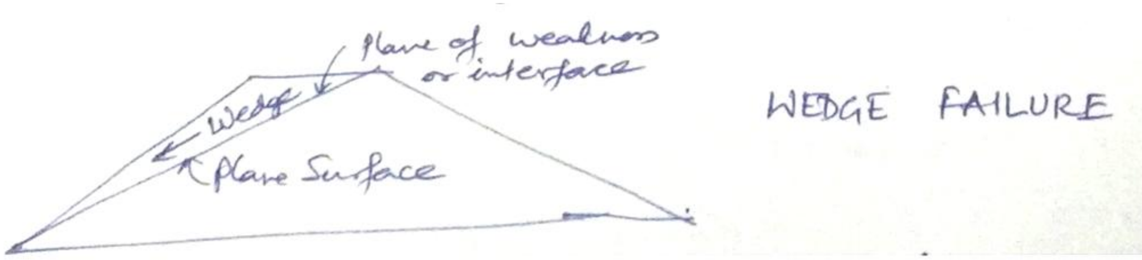
In practice, the slopes which are of considerable extent and in which the conditions on all vertical planes are adequately represented by average conditions are designated as infinite slopes.

3. Compound failure

- ➢ A compound failure is a combination of rotational and translational slip.
- ➢ A compound failure surfaces is curved at the two ends and plane in the middle portion.
- ➢ Compound failure generally occurs when a hard stratum exists at considerable depth below the toe.



4. **Wedge failure**



- A failure along an inclined plane is known as plane failure or wedge failure or block failure.
- It occurs when distinct blocks and wedges of soil mass become separated.
- A plane failure is similar to translational failure in many aspects.
- Translational failure occurs in infinite slopes whereas plane failure may occur even in finite slope consisting of two different materials or in a homogeneous slope having cracks, fissures, joints etc.

5. **Miscellaneous failures**

Complex failures such as spreads and flows.

INFINITE SLOPE

A constant slope of unlimited extent and having uniform soil properties at the same depth below the free surface is known as infinite slope.

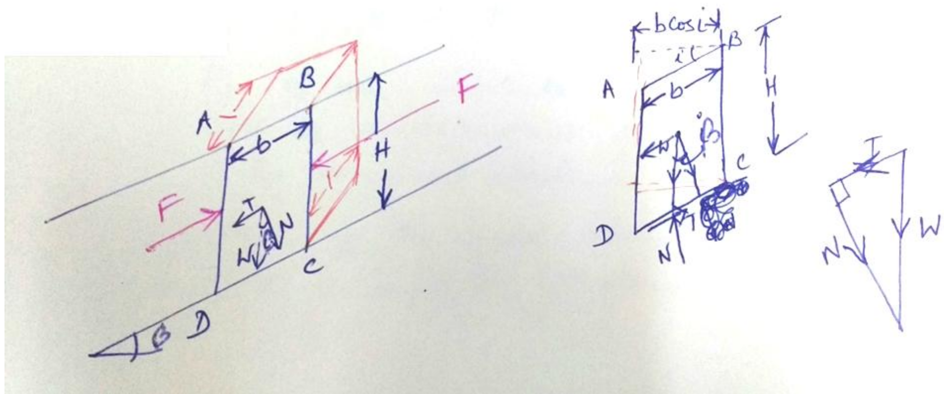
For an infinite slope in a homogeneous soil, the stresses and soil properties on every vertical plane are identical; on any plane parallel to the slope, they are again identical.

$\therefore$ Failure in such a slope takes place due to sliding of the soil mass along a plane parallel to the slope at a certain depth.

Stability analysis of an infinite slope of cohesionless soils

The stability criteria of an infinite slope of cohesionless soil will depend on whether the soil is (i) dry, (ii) submerged and (iii) has steady seepage.

1. **Dry soil :**



Volume of soil prism per unit length = Hb cos β

Weight of soil prism per unit length (W) = γ (Hb cos β)

Resolving W into normal component (N) and tangential component (T)

$$N = W \cos\beta = \gamma\, \mathrm{Hb} \cos^2\beta$$

Forces that tends to cause the slip along the plane →

$$T = W \sin\beta = \gamma\, \mathrm{Hb} \cos\beta \sin\beta$$

$$\text{Normal Stress, } \sigma = \frac{N}{b\,x\,1} = \gamma\, \mathrm{H} \cos^2\beta$$

Causes the slip / failure →

$$\text{Shear stress, } \tau = \frac{T}{b\,x\,1} = \gamma\, \mathrm{H} \cos\beta \sin\beta$$

The shear stresses τ tend to cause the shear failure along CD. This tendency is opposed by shearing resistance developed along the plane CD.

Shear strength, $s = c' + \sigma' \tan\Phi'$

As the soil is dry cohesionless, c′ = 0,

$$\therefore s = \sigma' \tan\Phi'$$

$$\text{or} \quad s = (\gamma\, \mathrm{H} \cos^2\beta) \tan\Phi'$$

The factor of safety against shear failure is given by

$$F_s = \frac{s}{\tau} = \frac{(\gamma\, \mathrm{H} \cos^2\beta)\tan\Phi'}{\gamma\, \mathrm{H} \cos\beta \sin\beta} = \frac{\tan\Phi'}{\tan\beta}$$

$$F_s = \frac{\tan\Phi'}{\tan\beta} - - - - - (1)$$

The above equation indicates that the slope is just stable when $\Phi' = \beta$ i.e., Fs = 1

The slope is stable when $\beta < \Phi'$

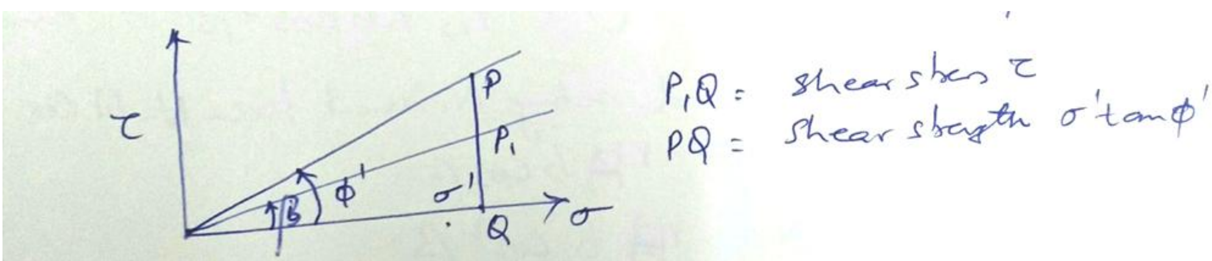
For slope angle, $\beta > \Phi'$, the slope is unstable.

Importance : Fs of an infinite slope of a cohesionless soil is independent of the height H of the assumed failure prism.

Φ' corresponding to loose state is generally taken as the soil in the surface layer is relatively loose.

Graphical representation of

$$F_s = \frac{\tan \Phi'}{\tan \beta}$$



2. Submerged slope

If the slope is submerged under water, the normal effective stress and the stresses are calculated using submerged unit weight.

$$\sigma' = \gamma' \text{ H} \cos^2\beta$$

$$\tau = \gamma' \text{ H } \cos\beta \sin\beta$$

γ' = submerged unit weight

$$F_s = \frac{s}{\tau} = \frac{(\gamma' \text{ H} \cos^2\beta) \tan \Phi'}{\gamma' \text{ H } \cos\beta \sin\beta}$$

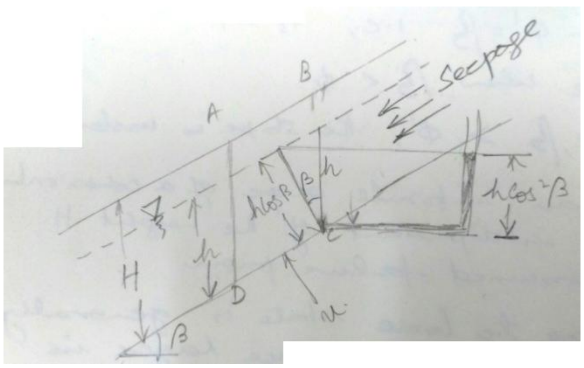
$$F_s = \frac{\tan \Phi'}{\tan \beta} \quad ------(2)$$

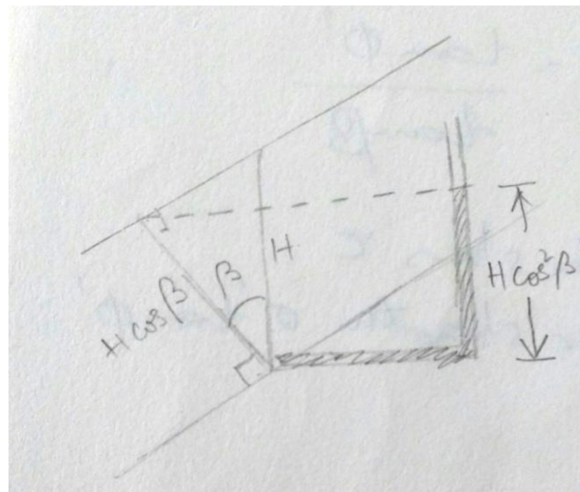
Comparing (1) and (2), Fs for submerged condition is equal to Fs in dry condition.

3. Stability of an infinite slope in cohesionless soil with seepage

With seepage taking place and the water table assumed to be parallel to the slope at a height h above the failure plane, pore water pressure on CD will be constant (i.e., water will rise to the same height in a piezometer inserted at any point on CD)

i.e., height of water column = $h\cos^2\beta$





u = upward pressure at the base = $\gamma_w\, h\cos^2\beta$

$\therefore$ Uplift force U = $\gamma_w\, h\cos^2\beta(\text{b x 1}) = \gamma_w\, hb\cos^2\beta$

Downward acting normal force $N = W\cos\beta$

$$\mathrm{W} = \gamma\mathrm{Hb}\cos\beta$$

$$N = \gamma\mathrm{Hb}\cos^2\beta$$

Net force acting downward $N' = N - U$

or $\quad N' = \gamma\mathrm{Hb}\cos^2\beta - \gamma_w\, hb\cos^2\beta = (\gamma\mathrm{H} - \gamma_w\, h)b\cos^2\beta$

Shear force or Tangential force $= W\sin\beta$

$$T = \gamma\mathrm{Hb}\cos\beta\,.\sin\beta$$

$$\therefore \sigma^{'} = \frac{N^{'}}{b} = (\gamma\mathrm{H} - \gamma_w\, h)b\cos^2\beta$$

$$\tau = \frac{N^{'}}{b} = \gamma\mathrm{H}\cos\beta\,.\sin\beta$$

Shear strength, $s = \sigma^{'}\tan\Phi' = (\gamma\mathrm{H} - \gamma_w\, h)\cos^2\beta\ \tan\Phi'$

$$F_s = \frac{s}{\tau} = \frac{(\gamma H - \gamma_w h)\cos^2\beta \tan\Phi'}{\gamma H \cos\beta . \sin\beta} = \frac{(\gamma H - \gamma_w h)\tan\Phi'}{\gamma H \tan\beta} = \left(1 - \frac{\gamma_w h}{\gamma H}\right)\frac{\tan\Phi'}{\tan\beta}$$

If water table is at ground surface, h = H and $\gamma = \gamma_{sat}$, then

$$F_s = \left(1 - \frac{\gamma_w}{\gamma_{sat}}\right)\frac{\tan\Phi'}{\tan\beta} = \frac{\gamma' \tan\Phi'}{\gamma_{sat} \tan\beta}$$

STABILITY ANALYSIS OF AN INFINITE SLOPE OF COHESIVE SOILS

Shear strength $s = c' + \sigma' \tan\Phi'$

(a) Dry soil

$$\sigma' = \gamma H \cos^2\beta \quad \& \quad \tau = \gamma H \cos\beta \sin\beta$$

$\therefore s = c' + (\gamma H \cos^2\beta) \tan\Phi'$

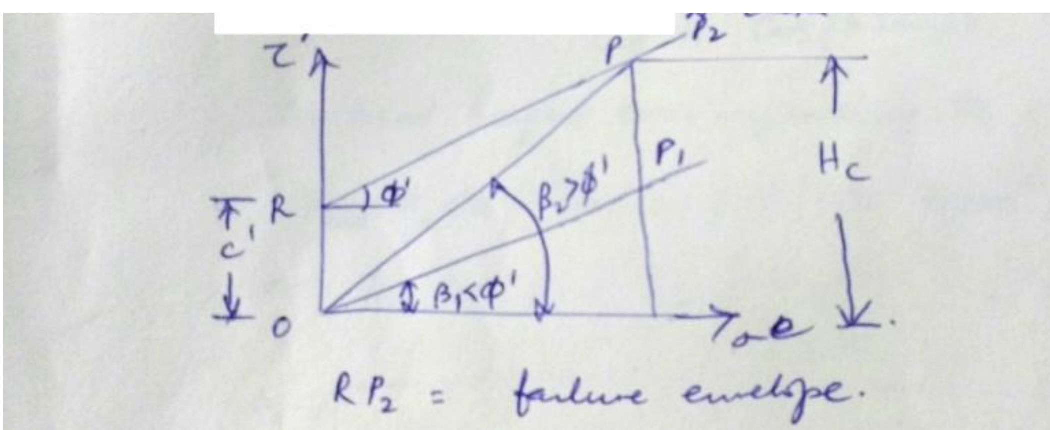
$\therefore$ The factor of safety Fs is given by

$$F_s = \frac{c' + (\gamma H \cos^2\beta) \tan\Phi'}{\gamma H \cos\beta \sin\beta}$$

← For heights less than critical height.

$\therefore$ The factor of safety of an infinite slope in cohesive soils depends not only on Φ' and β but also on γ , H and c'

Graphical method for determination of Factor of safety



RP_2 = Failure envelope

When β_1 is less than Φ' , the slope is always safe as shown by line OP_1.

When $\beta_2 > \Phi'$, the slope line cuts the failure envelope.
At point P_1, the slope is just stable.

For normal stress greater than that indicate by the point P_1 the shear stress is greater than shear strength and the slope is unstable.

Expression for height when the slope is just stable (Fs =1)

$$\gamma H \cos\beta \sin\beta = c' + (\gamma H \cos^2\beta) \tan\Phi'$$

$$\gamma H \cos^2\beta \left(\frac{\sin\beta}{\cos\beta} - \tan\Phi'\right) = c'$$

$$H_c = \frac{c'}{\gamma \cos^2\beta(\tan\beta - \tan\Phi')}$$

H_c is the height at which the slope is just stable.
H_c is known as critical height.

(b) Submerged slope

$$F_s = \frac{c' + (\gamma' H \cos^2\beta) \tan\Phi'}{\gamma' H \cos\beta \sin\beta}$$

For c - Φ soil

Φ′ should be taken corresponding to submerged condition.

(c) Steady seepage above the slope

$$F_s = \frac{c' + (\gamma' H \cos^2\beta) \tan\Phi'}{\gamma_{sat} H \cos\beta \sin\beta}$$

Critical height corresponding to $F_s = 1$

$$\gamma_{sat} H_c \cos\beta \sin\beta = c' + (\gamma' H_c \cos^2\beta) \tan\Phi'$$

$$H_c \cos^2\beta \left(\gamma_{sat} \frac{\sin\beta}{\cos\beta} - \gamma' \tan\Phi'\right) = c'$$

$$H_c = \frac{c'}{\cos^2\beta \left(\gamma_{sat} \tan\beta - \gamma' \tan\Phi'\right)}$$

$$H_c = \frac{c'}{\gamma_{sat} \cos^2\beta \left(\tan\beta - \frac{\gamma'}{\gamma_{sat}} \tan\Phi'\right)}$$

STABILITY ANALYSIS OF FINITE SLOPES

Finite slope is one with a base and top surface, the height being limited. The inclined faces of earth dams, embankments etc. are all finite slopes.

Investigation of the stability of finite slopes involves the following steps:

(a) Assuming a possible slip surface

(b) Studying the equilibrium of the forces acting on this surface.

(c) Repeating the process until the worst slip surface with minimum FOS is found.

Failure of finite slopes occurs along a curved surface.

Major classes of stability of analysis

1) Mass procedure
   - Mass of soil above sliding surface is taken as a unit
   - Useful when slope is assumed to be homogeneous
2) Method of slices
   - Soil above the sliding surface is divided into a no. of vertical parallel slices.

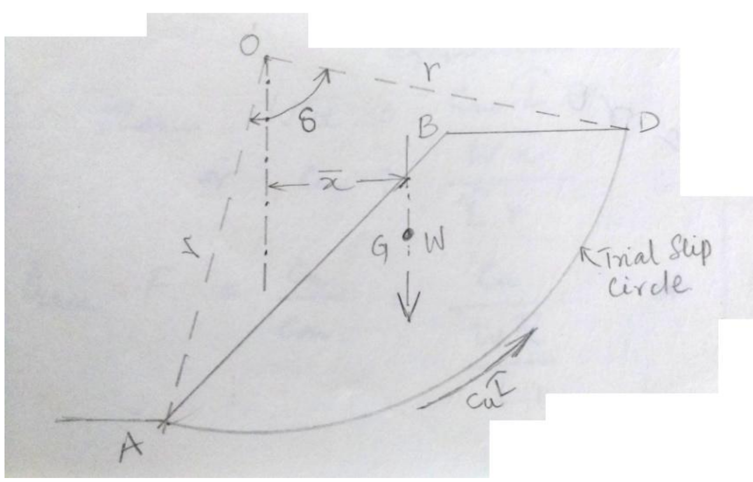
Φ_u = 0 Analysis

This method assumes that the surface of sliding is an arc of a circle. Two cases are considered.

Case 1. Total stress analysis for purely cohesive soils ($\Phi_u = 0$)

Case 2. Total stress analysis for c - Φ soil.

Case 1: **Φ_u = 0 (Homogeneous clay soil with undrained condition)**



- AB is the slope, the stability of which is to be determined.
- The method consists in assuming a number of trail slip circles, and finding the factor of safety of each.
- The circle corresponding to the minimum FOS is the Critical Slip Circle.

Let AD be the trail slip circle, with radius r and O as the center of rotation.

Let W be the weight of the soil of the wedge ABDA of unit thickness, acting through its centriod.

The driving moment $M_D = W\,\bar{x}$

Where $\bar{x}$ = distance of line of action of W from the vertical line passing through the center of rotation.

If c_u = unit cohesion
And $\hat{L}$ = length of slip arc AD
$\hat{L} = \frac{\pi}{180}\, r\, \delta$
δ measured in degrees

The shear resistance measured along the slip surface is $c_u\hat{L}$, which acts at a radial distance ‘r’ from the center of rotation O.

Hence the Resisting moment $M_R = r\, c_u\hat{L}$
The factor of safety

$$F = \frac{M_R}{M_D} = \frac{c_u\hat{L}.\ r}{W\,\bar{x}}$$

Alternatively,
Let c_m = mobilsed shear reissuance of soil ($\Phi = 0$) necessary for equilibrium.
Then

$$W\,\bar{x} = c_m\hat{L}.\ r$$

or

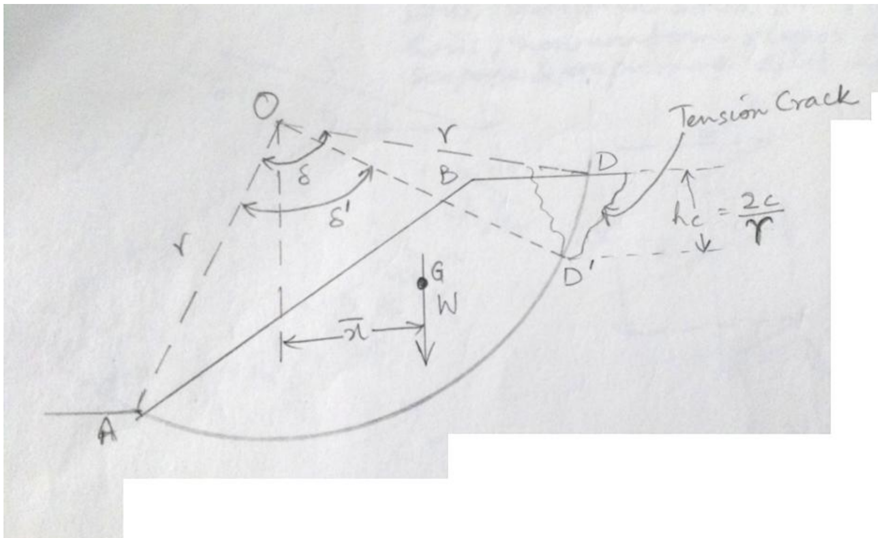
$$c_m = \frac{W\,\bar{x}}{\hat{L}.\ r}$$

Hence

$$F = \frac{c_u}{c_m} = \frac{c_u}{\frac{W\,\bar{x}}{\hat{L}.\ r}}$$

$$F = \frac{c_u\hat{L}.\ r}{W\,\bar{x}}$$

EFFECT OF TENSION CRACKS ON STABILITY



Note

- To calculate M_D use full length of arc AD
- To calculate M_R use reduced length of arc AD′ (use δ′)

In case of a cohesive soil, when the slope is on the verge of slippage there develops a tension crack at the top of the slope.
The depth of the tension crack is

$$h_c = \frac{2\,c}{\gamma}$$

Where γ = unit weight of soil.
There is no shear resistance along the crack. The failure arc reduces from AD to AD′, and the angle δ and δ′.

For computation of factor of safety

(i) Use δ′ instead of δ in the resisting moment M_R
(ii) Consider full weight 'W' of the soil within the sliding surface AD to compensate for filling of water in the crack, in the driving moment M_D.
Hence

$$F = \frac{c_u \widehat{L}\,.\,r}{W\,\bar{x}}$$

Where $\widehat{L} = \frac{\pi r \delta'}{180}$, δ' *measured in degrees.*

"Tension crack reduces FOS due to decrease in Resisting moment"

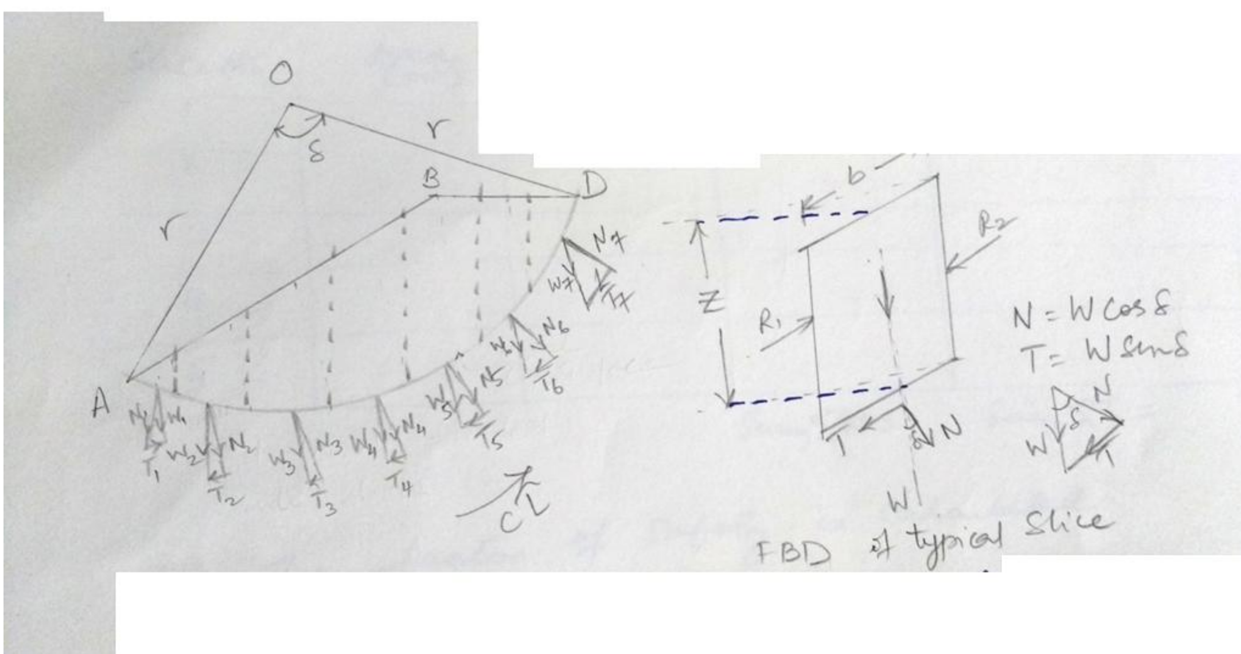
The effect of tension cracks are

(i) It modifies the slip surface and reduces length of slip surface.
(ii) It is usually filled with water and produces hydrostatic pressure along the depth.
(iii) It acts as channel for flow of water to underlying soil layers, inducing seepage forces

(iv) It reduces FOS.

SWEDISH METHOD OF SLICES FOR C - Φ SOIL

The method of slices is a general method which is equally applicable to homogeneous soils, stratified soils, fully or partly submerged soils, non-uniform slopes and to cases when seepage & pore pressure exist within the soil.



For a **c - Φ** soil the undrained strength envelope shows both c and Φ values. The total stress analysis is adopted.

The procedure is as follows.

1. Draw the slope to scale.
2. A trail slip circle such as AD with radius r is drawn from the center of rotation O.
3. Divide the soil mass above the slip surface into convenient no. of slices (more than 5).
4. Determine the area of each slice A_1, A_2,……A_n

   A = width of slice x mid height

   = b x z
5. Determine the total weight W including external load, if any, as

   $W = \gamma\, b\, z = \gamma\, A$ where γ = unit weight of soil

Note: The reactions R_1 and R_2 on the sides of the slice are assumed equal and therefore do not have any effect on stability.

6. The weight 'W' of the slice is set off at the base of the slice.

7. The values of $N = W \cos\delta \; and \; T = W \; \sin\delta$ are scaled off for each of the slices.
8. The values of 'N' and 'T' are tabulated as

Slice No.	Area (m^2)	Weight, W (kN)	Normal component, N (kN)	Tangential component, T (kN)
1				
2				
3				
4				
5				
			Sum ΣN =	Sum ΣT =

9. The factor of safety is calculate as follows
   The driving moment or sliding moment M_D is
   MD = r Σ T (taken positive if clockwise)

   Resisting moment or Restoring Moment M_R

$$M_R = r[c\,\Sigma\Delta\mathrm{L} + \tan\Phi\;\Sigma\mathrm{N}]$$

$$\Sigma\Delta\mathrm{L} = \frac{\pi r\delta}{180} = \text{length of the slip circle AD}$$

   Hence factor of safety against sliding is

$$F = \frac{M_R}{M_D} = \frac{c\,\hat{\mathrm{L}} + \tan\Phi\;\Sigma\,\mathrm{N}}{\Sigma\,\mathrm{T}}$$

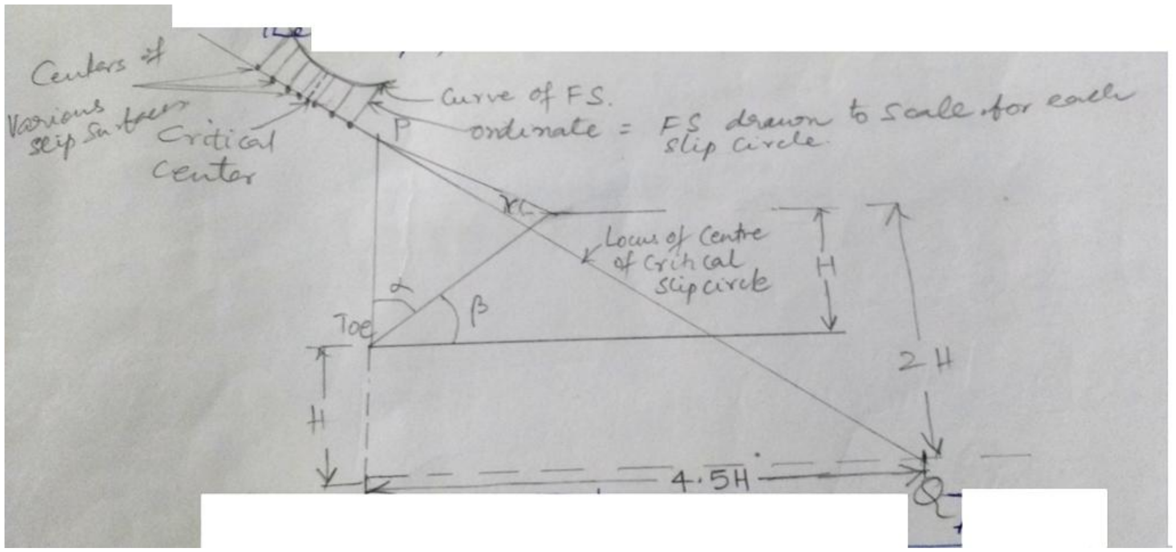
Note: The tangential components of a few slices at the base may cause restoring moments.

10. Repeat step 2 to 9 by considering various trail slip circles and calculate FS for each of these slip circles.
    The slip circle with a **minimum FS** is called "**Critical Slip Circle**"

Method of locating critical slip circle

In order to reduce the number of trails to find the center of critical slip circle, Fellinius has given a method of locating the locus on which the probable center may lie.

1. For a homogeneous c - Φ soil, the centers of slip circles lie on a line PQ, in which Q has its co-ordinates – H downwards from toe and 4.5 H horizontally away.
2. The other point P is located with the help of directional angles α and Ψ.



Slope	Slope angle β°	Directional Angles	
		α°	Ψ
0.58:1	60	29	40
1:1	45	28	37
1.5:1	33.8	26	35
2:1	26.6	25	35
3:1	18.4	25	35
5:1	11.3	25	35

3. When the line PQ is obtained, the trail centers are obtained on it and factor of safety corresponding to each centre is calculated.
4. The various factors of safety so obtained are plotted as ordinates from their respective centres.
5. A smooth curve is drawn through the ends of the ordinates.
6. The center corresponding to the lowest factor of safety is the center of the critical slip circle.

THE STABILITY OF AN EARTH DAM IS TESTED UNDER THE FOLLOWING CONDITIONS

1. Stability of d/s slope during steady seepage
2. Stability of u/s slope during sudden drawdown
3. Stability of u/s and d/s slopes during and immediately after construction.

1. Stability during steady seepage

When seepage occurs, pore water pressure (u) develops and this will reduce the effective stress which in further decreases the shear strength along the failure surface.

Adopt the following procedure to analyze the stability

(i) Draw the c/s of slope to scale

(ii) Draw the potential failure surface

(iii) Divide the soil mass between the slope and failure surface into slices of equal width.

(iv) Calculate the weight W and the corresponding normal and tangential components for all the slices.

(v) Construct the flownet for the embankment.

Pore pressure determination

(vi) The pore pressure at any point is represented by the piezometric head $\mathbf{h_w}$ at that point.

(vii) The variation of pore water pressure along a likely slip surface is obtained by measuring for entire slice the vertical height between the point of intersection of slip surface and an equipotential line, and the point where the equipotential line cuts the phreatic surface.

(viii) The average pore water pressure (u) at the bottom of each slice is given by the piezometric head (h_w) as

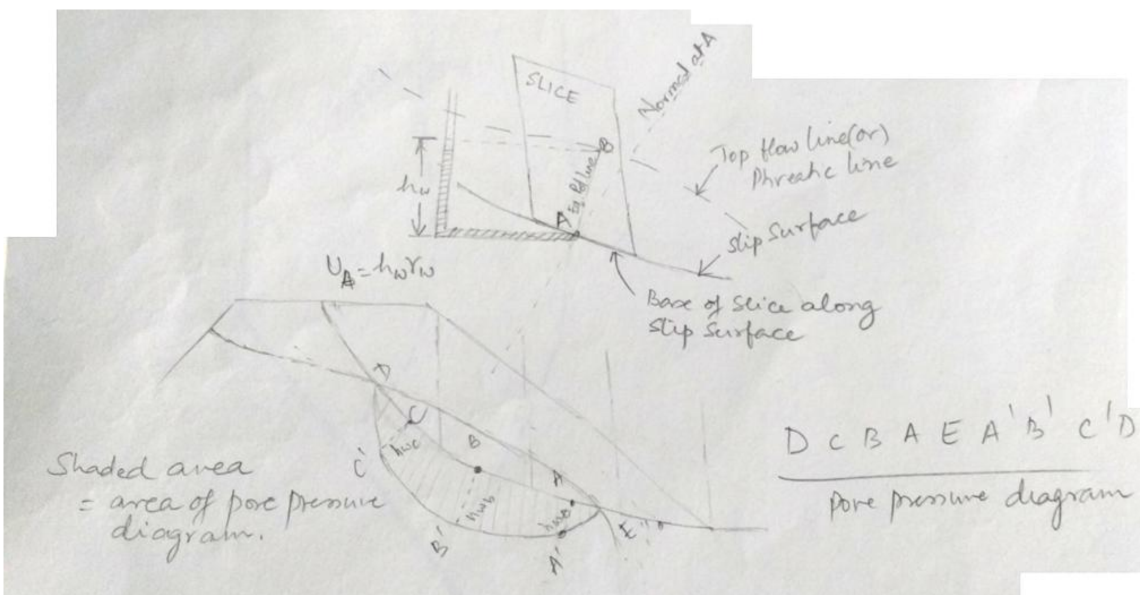
$$u = h_w \gamma_w$$

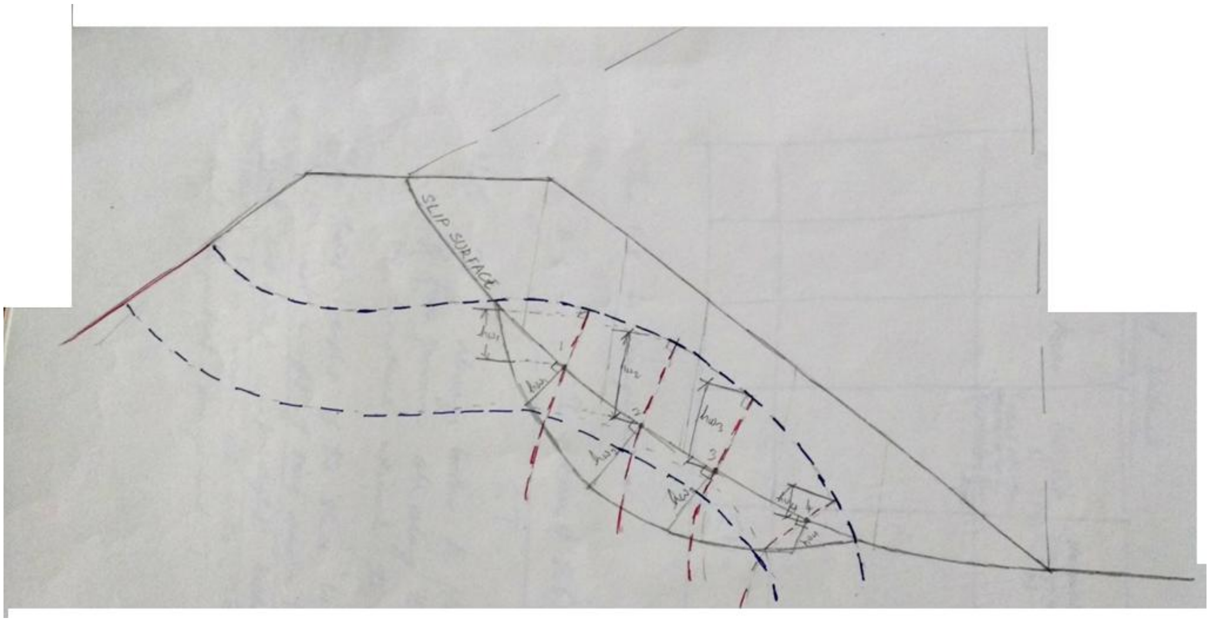
Where h_w = piezometric head above the base of the slice.

(ix) The force due to pore water pressure at the bottom of the slice is calculated as

$$U = u \,.\, A$$

Where A = area of pore pressure diagram.





Tabulated values

Slice No.	Width	Area	Weight, W Take γ above flow line & γ_{sat} below flow line	Normal component, N	Tangential component, T	Pore pressure (u)	Force due to pore pressure, (U)
				Σ N =	Σ T =		Σ U =

The FS is computed as

$$FS = \frac{c'\hat{L} + \tan\Phi' \Sigma (N - U)}{\Sigma T}$$

Note: It is always better to plot the variation of pore pressure all along the slip surface in directions normal to the slip circle.

The total weight of the slice 'W' is due to bulk unit weight of soil above the top of flow line and saturated unit weight below it. This weight W is resolved into normal component N and tangential component T.

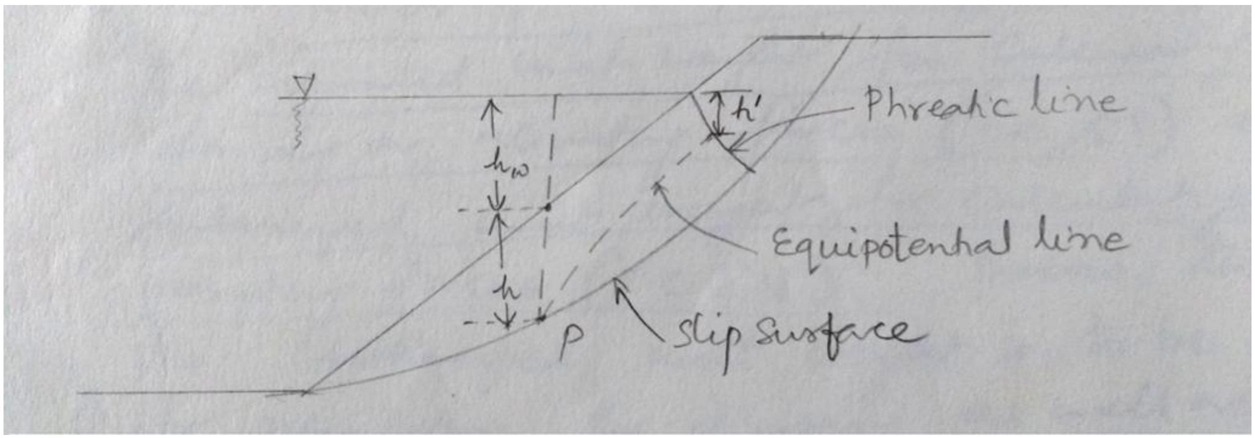
Stability of upstream slope during sudden drawdown

For the upstream slope (u/s), the critical condition is when the reservoir is suddenly emptied without allowing any appreciable change in the water level within the saturated mass of soil. This state is known as sudden drawdown.

During sudden drawdown of the reservoir, the direction of flow is reversed causing instability in the upstream slope of the earth dam.

With complete drawdown, hydrostatic force acting along the u/s slope at the time of full reservoir is emptied. The weight of water within the soil now tends to help in sliding failure without hydrostatic pressure on the slope to counter it.

For the analysis of sudden drawdown condition, the flownet corresponding to instantaneous drawdown is drawn. The pore water pressure distribution along the trial slip surface is determined.



The pore water pressure at any point P before drawdown is given by

$$u_0 = \gamma_w(h + h_w - h')$$

Where h = height of soil above P

h_w = height of water column above P

h′ = head loss due to seepage, indicated by the equipotential line passing through P.

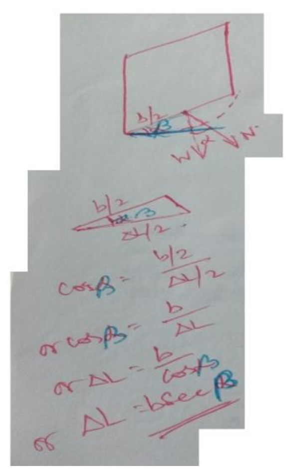
The pore water pressure at P immediately after the drawdown is given by

$$u = \gamma_w(h - h')$$

Using the above expression, pore water pressure (u) at various points on the slip surface is determined.

The factor of safety is obtained from

$$F_s = \frac{\Sigma c\ \Delta L + \ \Sigma(N - ul\ ) \tan \Phi}{\Sigma T}$$



$$l = \Delta L = b \sec \beta = curved\ length\ of\ the\ base\ of\ the\ slice$$

β = slope angle

$$\cos \beta = \frac{b/2}{\Delta L/2} = \frac{b}{\Delta L}$$

$$\therefore \Delta L = \frac{b}{\cos\beta} = b\sec\beta$$

APPROXIMATE METHOD

The factor of safety is computed by considering the saturated unit weight for calculating the driving or actuating forces (i.e., Σ T) and submerged unit weight for calculating the resisting forces (i.e., ΣN). Below the drawdown, the submerged unit is to be used for calculating the driving as well as the resisting forces.

STABILITY DURING AND AT THE END OF CONSTRUCTION

An embankment dam is normally compacted at 80 to 90% saturation i.e., 80 to 90% pore space is filled by water and the rest by air.

The computation of this water air pore fluid under increasing load of embankment causes the buildup of pore pressure in fine grained soils.

The magnitude of pore pressures depends on several factors such as placement of water content, compressibility and permeability of the fill material, and rate of construction.

Prediction of pore pressures during the design stage is difficult. Hence construction pore pressures are estimated based on the results of laboratory tests.

When the soil is not consolidated to complete saturation, pore pressure can be determined using Hilf's equation.

$$u = \frac{p_a \Delta}{V_a + h_c V_w - \Delta} - - - - - - (1)$$

Where p_a = air pressure in the voids after initial compaction.

$$\Delta = \frac{\Delta e}{1 + e_0}$$

Δe = change in void ratio
e_0 = initial void ratio
V_a = Volume of air after initial compaction (%)
V_w = Volume of water after initial compaction (%)
h_c = Henry's constant of soluibility of air in water = 0.02 at 20°C

Consolidation tests are conducted on soil samples. A plot is made between effective stress σ′ and percent consolidation, Δ. For different values of Δ, u is obtained from eqn. (1) above.

Total stress σ is given by
σ = σ′ + u
A plot is then made between σ and u.

Total stress at any point = Bulk unit weight x depth of soil above.
This point is used for the determination of pore pressures at various points in the dam during construction.

FRICTION CIRCLE METHOD

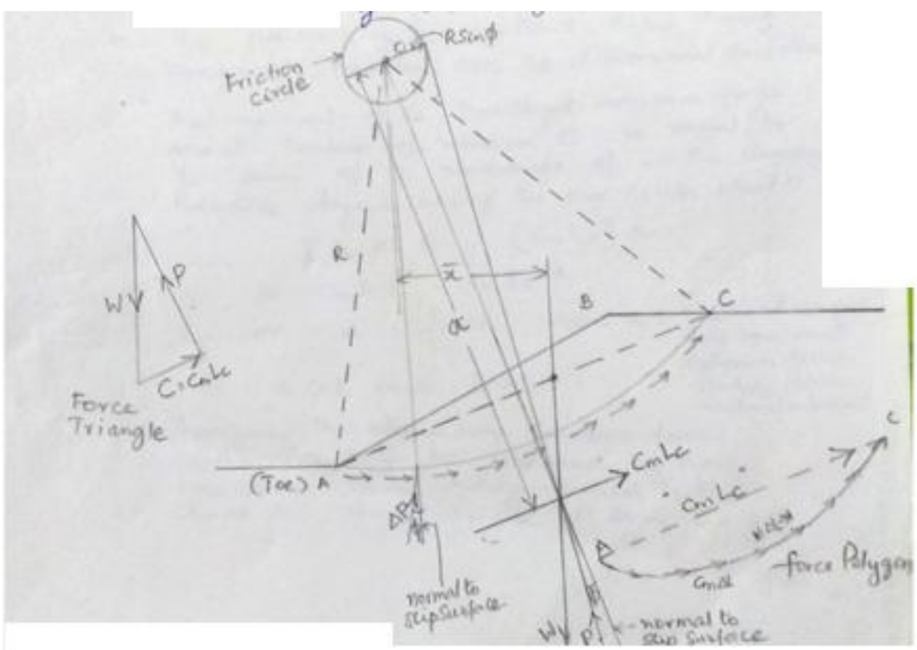
Summary

This method uses total stress based limit equilibrium approach. In this method, the equilibrium approach. In this method, the equilibrium of the resultant weight 'W', the reaction 'P' due to frictional resistance and cohesive force 'C' are considered.

The magnitude, direction and line of action of 'W', the lines of action of the reaction force 'P' and cohesive force 'C' being known, the magnitude of P and C are determined by considering the triangle of forces. The FS w.r.t. cohesion and friction is evaluated.

Procedure

1. The slope is drawn to scale.
2. A trail circular slip surface of radius R with 'O' as center is drawn from the toe.
3. The centroid of the sliding mass ABCA and weight of the sliding mass 'W' is determined.



Resultant cohesive force and its point of application

4. Let the length of the slip circle arc AC be 'L'
5. Let the slip circle be considered to be made up of a number of elementary arcs, each of length 'ΔL'
6. The elementary cohesive force acting along an element of length ΔL opposing the probable movement of the soil mass is $c_m \Delta L$.

c_m = unit mobilised cohesion, assumed constant along the slip surface.

7. The total cohesive force is considered to be made up of elementary cohesive forces c_m ΔL, representing a force polygon.
8. The closing link of the force polygon represents the magnitude and direction of the resultant cohesive force.
9. If 'L_c' is the length of the chord AC, the magnitude of the resultant cohesive force is c_m L_c
10. The position of the resultant c_m L_c using Varignon's theorem, can be determined as follows:
    "The moment of the resultant cohesive force about center of rotation 'O' is equal to the sum of the moments of all the elementary cohesive forces along the slip circle about 'O'.

$$\therefore\ \Sigma c_m \Delta R = (c_m L_c)\ a$$
$$\text{or}\quad c_m\, R\, L = c_m L_c\, a$$
$$\text{or}\quad a = \frac{L}{L_c}\,.R$$

Where a = moment arm of the resultant cohesive force $c_m L_c$, from centre of rotation O.
a > R since L>L_c

Therefore, the elementary cohesive c_m ΔL can now be replaced by their resultant "$c_m L_c$" acting parallel to the chord AC at a distance 'a' from 'O'.

<u>Reaction 'P' due to frictional Resistance</u>

11. If it is assumed that the frictional resistance is fully mobilised along the slip surface, the soil reaction ΔP on any elementary arc has its direction opposing the probable movement of sliding mass.
12. ΔP is inclined at angle Φ to the normal at the point of action of ΔP on the slip circle.
13. The line of action of ΔP, and also the resultant reaction P will be tangential to a circle of radius of $R \sin \Phi$ drawn with 'O as center. This circle is called "friction circle".
14. The three forces considered in the equilibrium of sliding soil mass can now be drawn.
    (i) Weight 'W' is drawn as a vertical line passing through the centriod of area ABCA.
    (ii) The resultant cohesion C_m L_c is drawn parallel to chord AC at a distance 'a' from center 'O'.
    (iii) The resultant reaction 'P' has to pass through the point of intersection of 'W' and "C_m L_c" and will be tangential to friction circle.
15. Knowing the magnitude of 'W' and direction of lines of action of 'W' , 'P' and 'C', the force triangle is drawn.
16. The unit mobilised cohesion 'C_m' required for equilibrium is determined from the force triangle as $c_m = \frac{C}{L_c}$

    FS w.r.t cohesion is determined as $F_c = \frac{c}{c_m}$

    The minimum factor of safety is obtained by locating the slip circle (Fellinius Method)

Computation of FS w.r.t strength

1. Assume a trail FS w.r.t friction as F_Φ
2. Draw a friction circle with a reduced radius $R \sin \Phi_m$

Where $\tan \Phi_m = \frac{\tan \Phi_u}{F_\Phi}$

3. Carry out the friction analysis and find FS w.r.t cohesion

$$F_c = \frac{C_u}{C_m}$$

4. If $F_c = F_\Phi$, it represents FS w.r.t strength
5. Else, choose different F_Φ and repeat the procedure till $F_c = F_\Phi$.

STABILITY ANALYSIS OF FINITE SLOPES WITH TAYLOR'S CHARTS.

Based on the principle of determining Fc or Fs for infinite slope, Taylor established a relationship between the three variables

(i) Stability number, $S_n \left(= \frac{C_m'}{\gamma H}\right), \left(c_m = \frac{c'}{F_c}\right)$

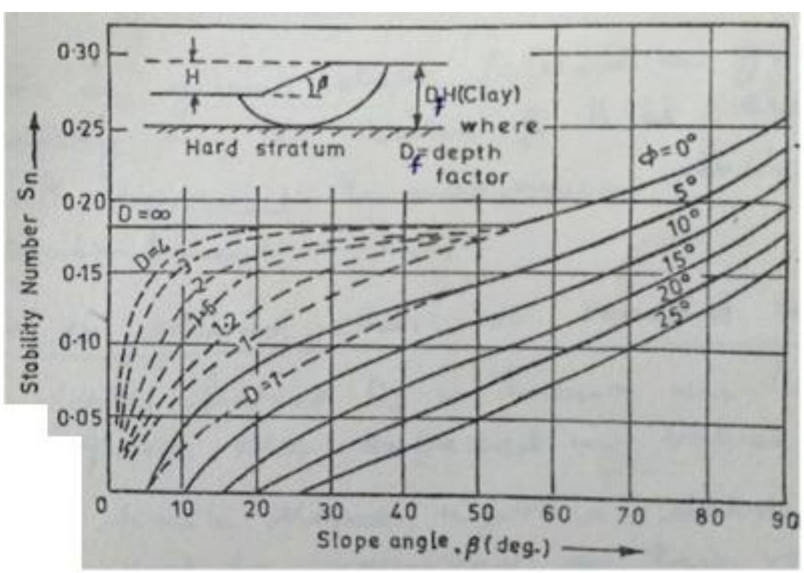
(ii) Angle of internal friction, Φ

(iii) Slope angle, β

The relation is expressed in the form of ready-made charts, called Stability charts.

The first set of curves, in dark lines is plotted for simple finite slope of height H between the stability number S_n, and inclination of the slope with the horizontal β, with Φ as the third parameter.

1. **Darklines** First chart or first set of curves.
2. Most critical circle passes through toe of slope.
3. Φ > 0°
   Toe failure occurs when Φu > 0 or Φu = 0 but β > 53°



1. **Dotted lines** Second set of curves or second chart.
2. **Φ = 0**
3. Most critical circle goes to an infinite depth.
   Base failure when Φu = 0 but β < 53°

Different curves between Sn and β are drawn for different values of Φ.

The first set of curves (or first chart) is based on the position that the most critical circle passes through the toe of the slope.
However, for β > 53° and for small values of Φ (upto 5°) a more critical circle may pass below the toe.

In the second set of curves shown in dotted lines, Φ is taken as zero, such as in the case of undrained saturated clays. In such cases, the most critical circle theoretically goes to an infinite depth. However, in practical, its depth will be limited by the presence of a hard stratum available at a depth D_f H, below the top of the slope of height H.

Use of first chart or first set of curves (Dark lines, Φ > 0°)
The first chart can be used in two ways:

1. For a slope of a given inclination β made in a soil of known Φ, with embankment height H, Sn is first read for a given β and Φ.

   $$S_n = \frac{C_m}{\gamma H}$$

   Knowing the value of Sn, Cm is determined as $C_m = S_n \gamma H$

   The factor of safety w.r.t cohesion is finally determined as

   $$F_C = \frac{C'}{C_m}$$

2. For a given slope β and a given factor of safety F_c, the value of H is evaluated, which will represent the maximum stable height of the embankment or cut.

Use of second chart or second set of curves (Dotted lines, Φ = 0°)

The depth factor D_f is known in this case. These stability are also expressed in the values in tables.

Important note

For finite slopes, rigorous stability analysis may not be required, as their can be examined easily with help of these charts.

For submerged slopes, the stability number Sn (= Cm / γH) can be calculated using γ' for γ ; and angle of friction Φ as equal to Φ_m if Fs is to be accounted for , or Φ if Fc is to be accounted for.

Under sudden drawdown conditions, Sn (= Cm / γH) is calculated using γ_{sat} for γ. The value of Φ also gets reduced in the ratio γ'/γ_{sat} . In such a case, this value of Φ is called weighted friction angle (Φw) and $\Phi = (\gamma'/\gamma_{sat}) \times \Phi_m$; Φ to be used against Φ_m is Fc is accounted for.

FORMULAE

1. $F_s = \frac{S}{\tau_m} = \frac{C' + \sigma' \tan \Phi'}{c_m + \sigma' \tan \Phi_m}$

$$C_m = \frac{C}{F_s}, \qquad \tan \Phi_m = \frac{\tan \Phi'}{F_s}$$

2. $F_c = \frac{C}{C_m}$
3. $F_\Phi = \frac{\tan \Phi'}{\tan \Phi_m}$

Infinite slope – cohesionless soil

(a) Dry: $F_s = \frac{\tan \Phi'}{\tan \beta}$

(b) Submerged: $F_s = \frac{\tan \Phi'}{\tan \beta}$

With seepage

(c) W.T at ‘h’ above failure surface

$$F_s = \left(1 - \frac{\gamma_w h}{\gamma H}\right) \frac{\tan \Phi'}{\tan \beta}$$

(d) W.T at ground surface

$$F_s = \left(1 - \frac{\gamma_w}{\gamma_{sat}}\right) \frac{\tan \Phi'}{\tan \beta}$$

Infinite slope – cohesive soils

(a) Dry soil:

$$F_s = \frac{C' + (\gamma H \cos^2 \beta) \tan \Phi'}{\gamma H \sin \beta \cos \beta}$$

Critical height $H_c = \frac{C'}{\gamma \cos^2 \beta (\tan \beta - \tan \Phi')}$

(b) Submerged: $F_s = \frac{C' + \gamma' H \cos^2 \beta \tan \Phi'}{\gamma' H \sin \beta \cos \beta}$

Critical height $H_c = \frac{C'}{\gamma' \cos^2 \beta (\tan \beta - \tan \Phi')}$

(c) Seepage:

$$F_s = \frac{C' + \gamma' H \cos^2 \beta \tan \Phi'}{\gamma_{sat} H \sin \beta \cos \beta}$$

$$\text{Critical height } H_c = \frac{c'}{\gamma_{sat} \cos^2 \beta \left(\tan \beta - \frac{\gamma'}{\gamma_{sat}} \tan \Phi' \right)}$$

Finite slope

$$F_s = \frac{c_u \hat{L} . r}{W \bar{x}}$$

Where $\bar{L} = \frac{\pi}{180} . r\delta$

When dry tension crack exists,

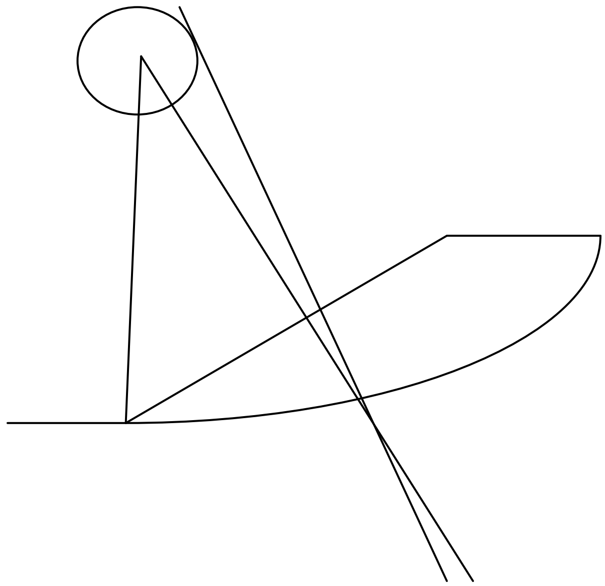
Take reduced $\bar{L}'(= AD)$
Reduced $W' = \gamma(A - A')$

When tension crack is filled with water

$$F_s = \frac{c_u \hat{L} . r}{W' \bar{x} + P_w l}$$

Finite slope analysis

Friction circle method (for c-Φ soil)



Objective: To find factor of safety
Circular slip surface is assumed.
We know that the soil frictional in nature, which means that when the soil mass in ABCA tries to move along the slip surface, because of the friction there will be frictional resistance (P). Here P is the resultant frictional resistance.

According to this method, the line of action of frictional resistance is tangential to a circle whose radius is " $\sin \Phi$ " and this circle is called the friction circle. The friction circle is concentric with circular slip surface.

$$FOS = \frac{\tau}{\tau_d}$$

$$\tau_d = \frac{\tau}{FOS}$$

$$\tau_d = \frac{c_d^{'} + \sigma' \tan \Phi_d^{'}}{FOS}$$

$$\tau_d = \frac{c}{FOS} \quad , \quad \tan \Phi_d^{'} = \frac{\tan \Phi^{'}}{FOS}$$

First we assume a factor of safety for internal angle of friction and then find Φ_d

$$\tan \Phi_d' = \frac{\tan \Phi'}{FOS}$$

Next draw friction circle is drawn with a radius $\boldsymbol{r \sin \Phi}$

Then find F_c ,

$$F_c = \frac{c}{c_d}$$

If the calculated F_c is equal to F_Φ, then our calculation is correct. Else assume another F_Φ and repeat the procedure.

Consider the equilibrium of three forces

1. Weight of soil above the slip surface – magnitude and direction
2. Resultant cohesive force – direction
3. Resultant frictional force – direction.

Total cohesive force $C = C_d L_c$, $C_d = C / L_c$

Consider elementary arc of length ΔL

Mobilised cohesion = C_d

Cohesive force on elementary arc = $C_d \Delta L$

Draw force polygon for elementary cohesive forces.

The closing side of force polygon of elementary cohesive forces gives the magnitude and direction of resultant force.

Length of the closing side = L_c = Chord length

Direction of cohesive force = direction of chord AC

Total cohesive force = $C_d L_c$

Location of cohesive force,

Consider moment of elementary forces

Σ of moment of elementary forces about center = moment of resultant force about center

Resultant force = $C_d L_c$ at a distance ‘a’ from center

$$\Sigma(\Delta C \, \Delta L) R = C_d L_c \cdot a$$

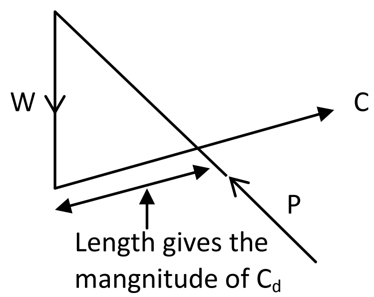
$$\Sigma \Delta L = \text{total length of arc} = L$$

$$C_d\, L\, R = C_d\, L_c\, .\, a$$

$$a = \frac{L}{L_c} R \qquad\qquad L_{arc} > L_{chord}$$

Through the CG of the slope section, draw the weight W. Draw C_d L_c at ‘a’ distance. Resultant frictional force P should pass through the point of intersection of W and C_d L_c for equilibrium.

∴ Resultant frictional force should pass through the point of intersection of C and W, and also be tangential to friction circle.



W is drawn to scale vertically.

Line of action of P is drawn to cut line of action of W.

Then line of action of C is drawn.

Length of the segment gives magnitude C_d

$$C = C_d\, .\, L_c$$

$$Mobilised\ cohesion, C_d = \frac{C}{L_c}$$

$$\text{Factor of safety for cohesion, } F_c = \frac{C}{C_d}$$

If the above $F_c = F_\Phi$, then our analysis is correct or else repeat the procedure.

Note:

Φ is the angle between normal and resultant