Unit – II

# EARTH PRESSURE THEORIES

## INTRODUCTION

Soil mass is stable when the slope of the surface of soil mass is flatter than safe slope. A slope steeper than safe slope may have to be provided in places where there is space restriction. In such cases a retaining structure may be required to provide lateral support to the soil mass. A retaining wall maintains the soil at different elevations on either side.

Design of retaining structure requires the determination of magnitude and direction of line of action of lateral earth pressure. The magnitude of lateral earth pressure depends on several factors such as

a) Mode of movement of the wall
b) Flexibility of the wall
c) Properties of the soil
d) Drainage conditions

Lateral earth pressure is determined using the theories proposed by Coulomb (1773) and Rankine (1857).

## Types of lateral earth pressure

Depending on the movement of the retaining wall with respect to the soil retained, lateral earth pressure is grouped in to three categories.

1. At rest pressure

   The lateral earth pressure is called at-rest pressure when the soil mass is not subjected to any lateral yielding or movement. This case occurs when retaining wall is firmly fixed at its top and is not allowed to rotate or move laterally. At-rest pressure is also known as the equilibrium condition, as no part of the soil mass has failed and attained the plastic equilibrium.

   Ex. Basement retaining walls fixed at the top with basement slab, bridge abutment wall fixed at the top with bridge slab.

2. Active earth pressure

   A state of active pressure occurs when the soil mass yields in such a way that its tend to stretch the soil mass horizontally. It is a state of plastic equilibrium as the entire soil mass is on the verge of failure. A body of soil is said to be in plastic equilibrium if every point of it is on the verge of failure.

   Active earth pressure is the lateral earth pressure exerted on the retaining wall when the wall moves away from the backfill (soil retained by the retaining wall).

   When the wall moves away from the backfill, a wedge shaped portion of the backfill located immediately behind the wall, known as failure wedge or sliding wedge, breaks away from the rest of the soil mass moves downwards and outwards.

In active case, the lateral pressure exerted on the wall is minimum. The soil is on the verge of failure due to decrease in lateral stress. The horizontal strain required to reach active state of plastic equilibrium is as small as 0.5%.

A state of shear failure corresponding to the minimum lateral earth pressure is the active state.

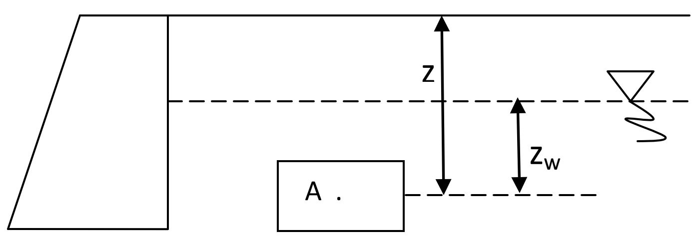
3. Passive earth pressure
   A state of passive pressure occurs when the soil mass yields in such a way that its tends to compress the soil mass horizontally. It is a state of plastic equilibrium as the entire soil mass is on the verge of failure.

   Passive earth pressure is the lateral earth pressure exerted on the retaining wall when the wall moves towards the backfill.

   In passive case, the failure wedge or sliding wedge moves upwards and inwards. The soil is on the verge of failure due to an increase in the lateral stress. A much larger horizontal strain of 5% in case of dense sands and 15% in case of loose sands is required to reach full passive earth pressure.

   A state of shear failure corresponding to the maximum lateral earth pressure is the passive state.

**Earth pressure at rest:** This is a special case of elastic equilibrium. The state of stress corresponds to the condition when there is no movement.




The vertical effective stress at point A at a depth z is
$\sigma'_z = \gamma z - \gamma_w z_w$ (total stress = $\gamma z$ , pore water pressure = $\gamma_w z_w$)
The coefficient of earth pressure at rest $k_0$ is given by

$$K_0 = \frac{\sigma_x'}{\sigma_z'}$$

Or

$$\sigma_x' = K_0 \sigma_z' = K_0(\gamma z - \gamma_w z_w)$$

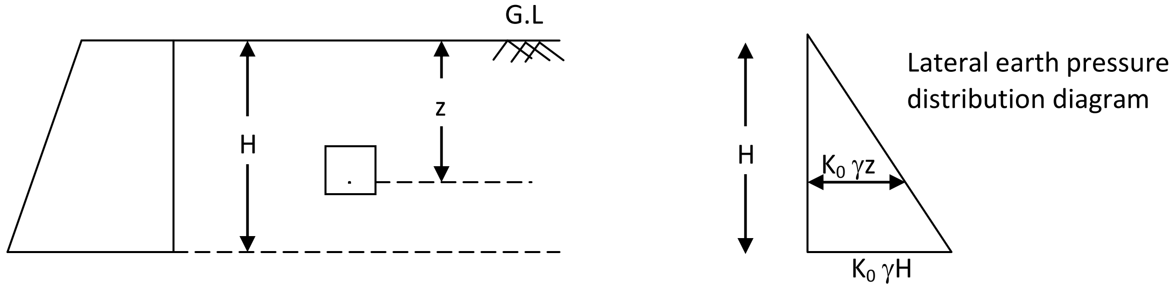
$\sigma_x'$ is usually represented by $p_0$ , indicating lateral earth pressure at rest.
$\therefore\ p_0 = \sigma_x' = K_0 \sigma_z'$

Or $p_0 = K_0(\gamma z - \gamma_w z_w)$

The lateral pressure $p_h$ is the sum of lateral stress ($p_0$) and pore water pressure (u)

$$\therefore p_h = p_0 + u = K_0(\gamma z - \gamma_w z_w) + \gamma_w z_w$$

**Earth pressure at rest – Dry condition**




The pressure at the bottom of the wall at depth z is given by

$$p_h = K_0 \gamma z$$

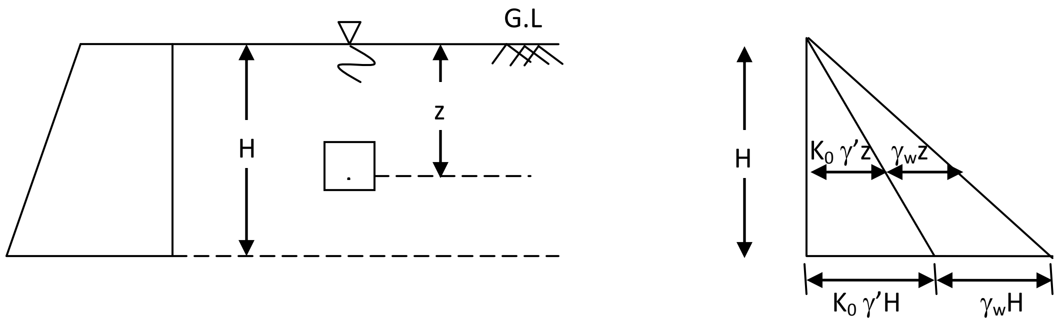
At a depth H

$$p_h = K_0 \gamma H$$ ← Dry Condition

The pressure distribution is thus triangular. The total force per unit length of wall is thus

$$P_h = \int_0^H K_0 \gamma z\, dz = \frac{1}{2} K_0 \gamma H^2 = Area\ of\ pressure\ distribution\ diagram$$

**Earth pressure at rest – Water table at ground surface**




Lateral earth pressure distribution diagram

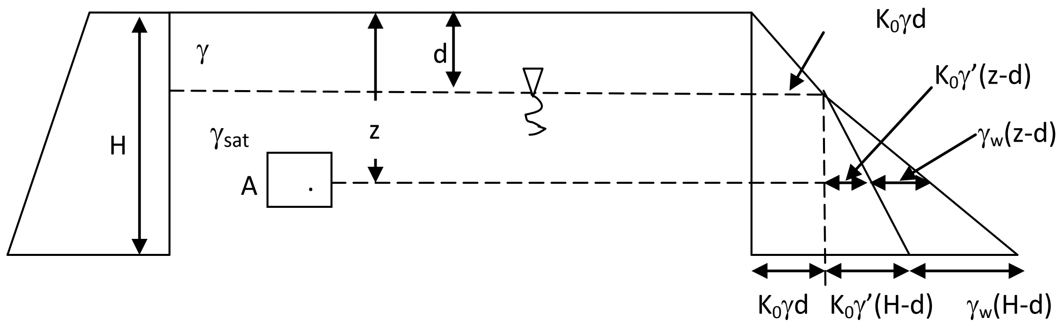
Effective vertical stress $\sigma_z' = \gamma_{sat} H - \gamma_w H = H(\gamma_{sat} - \gamma_w) = H\gamma'$

Effective lateral stress, $p_0 = k_0 \sigma_z' = k_0 \gamma' H$

Total lateral earth pressure = $p_h = p_0 + u$

$$\therefore p_h = k_0 \gamma' H + \gamma_w H$$ ← WT at ground surface

**Earth pressure at rest – WT at some depth 'd' below GL**




Total vertical stress $\sigma_z$ at A is

$\sigma_z = \gamma d + \gamma_{sat} (z - d)$

Pore water pressure 'u' at A is

$u = \gamma_w (z - d)$

$\therefore$ Effective vertical stress at A, $\sigma'_z$ is

$\sigma'_z = \sigma_z - u$

or $\sigma'_z = \gamma d + \gamma_{sat} (z - d) - \gamma_w (z - d)$

or $\sigma'_z = \gamma d + ( \gamma_{sat} - \gamma_w ) (z - d)$

or $\sigma'_z = \gamma d + \gamma'(z - d)$

Effective lateral stress at A is

$\sigma_x' = p_0 = K_0 \sigma_z'$

or $p_0 = K_0[\gamma d + \gamma'(z - d)]$

Total lateral earth pressure $p_h$ is

$p_h = p_0 + u$ , where $u = \gamma_w(z - d)$

$\therefore p_h = K_0[\gamma d + \gamma'(z - d)] + \gamma_w(z - d)$

If the water table is at the ground surface, d = 0

$\therefore p_h = K_0[0 + \gamma' z] + \gamma_w z$

or $p_h = K_0 \gamma' z + \gamma_w z$

At depth H, the lateral earth pressure $p_h$ is

$p_h = K_0 \gamma' H + \gamma_w H$

**Rankine's theory of earth pressure**
Rankine's theory considers the stress in a soil mass when it reaches a state of plastic equilibrium. General states of plastic equilibrium, where the entire mass of soil comes under plastic equilibrium are seldom realized in practice.

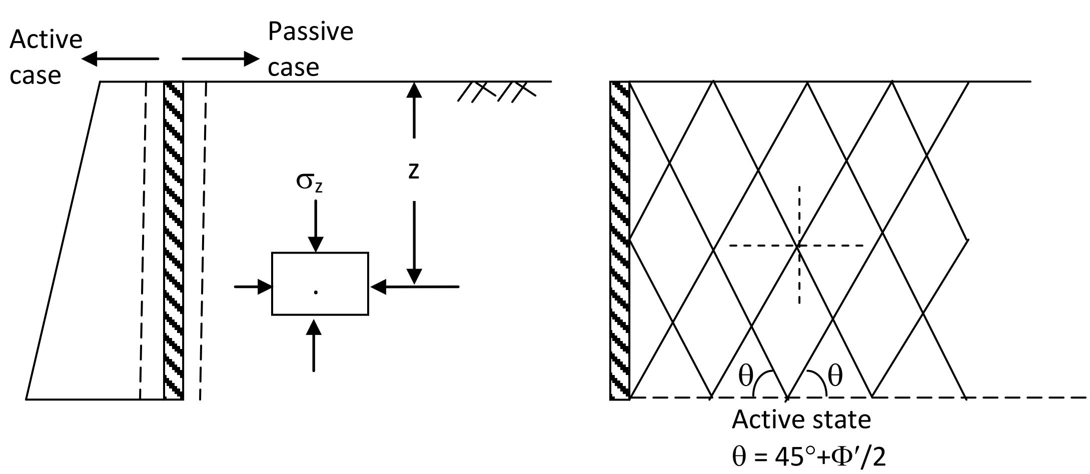
In practical situations, plastic equilibrium conditions are realized in limited zones of soil and they are realized in limited zones of soil and they are known as local states of plastic equilibrium.

The development of earth pressure against a retaining wall is a consequence of a portion of the backfill coming under plastic equilibrium.

**Assumptions in Rankine's theory**

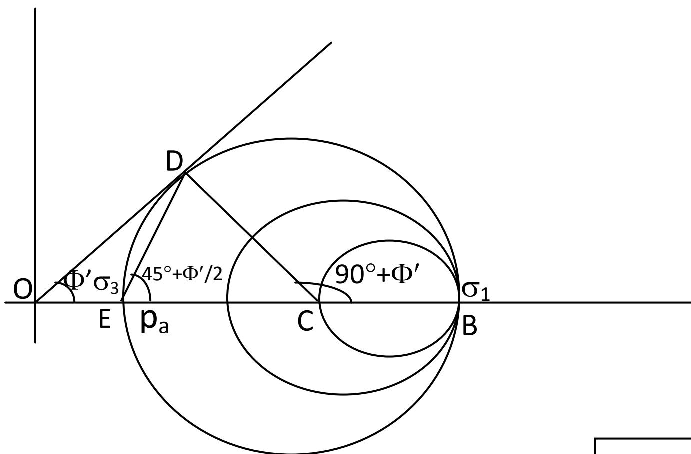
1. The soil mass is semi-infinite, homogeneous, dry and cohesionless.
2. The ground surface is a plane which may be horizontal or inclined.
3. The backfill of the wall is vertical and smooth. This implies that there are no shearing stresses between the wall and the soil and the stress relationship for any element adjacent to the wall is the same as for any other element far away from the wall.
4. The wall yields about the base and thus satisfies the deformation condition for plastic equilibrium.

**Comments on assumptions**
Retaining walls are constructed of masonry or concrete. Hence, back of the wall is never smooth. Due to this, frictional forces develop. Consequently, the resultant pressure (or reaction) will not be parallel to the surface of the backfill (or normal to the vertical wall surface). The existence of friction makes the resultant pressure inclined to the normal at angle that approaches friction angle between soil and wall.



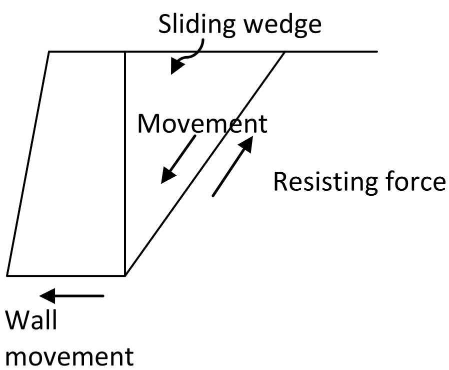

**Active state**
Consider a soil element at a depth z subjected to vertical stress $\sigma_z$ and a horizontal stress. Since no shear stresses act on the horizontal and vertical planes $\sigma_z$ and $\sigma_x$ are the principal

stresses. If the wall moves away from the backfill, the soil element expands and the value of $\sigma_x$ decreases. If the expansion is large enough, $\sigma_x$ decreases to a minimum value so that a state of plastic equilibrium is developed. At this stage, the Mohr circle representing the stressed state touches the strength envelope of the soil. Since this state is developed by a decrease in the value of $\sigma_x$, it must be minor principal stress, $\sigma_3$. The vertical stress $\sigma_z$ is the major principal stress, $\sigma_1$. As the soil stretches, the vertical stress $\sigma_z$ (or $\sigma_1$) will remain the same, since it is the weight of the soil above the horizontal plane.




OE = $\sigma_3$
DC = EC = radius of Mohr circle
$= \frac{\sigma_1 - \sigma_3}{2}$

$$oc = \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2}$$




From triangle OCD

$$\sin \Phi' = \frac{DC}{OC} = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2}$$

or $(\sigma_1 + \sigma_3) \sin \Phi' = \sigma_1 - \sigma_3$

or $\sigma_1 \sin \Phi' + \sigma_3 \sin \Phi' = \sigma_1 - \sigma_3$

or $\sigma_1 (1 - \sin \Phi') = \sigma_3 (1 + \sin \Phi')$

or $\frac{\sigma_1}{\sigma_3} = \frac{(1+\sin \Phi')}{(1-\sin \Phi')}$

$$\sigma_3 = \sigma_1 \frac{(1 - \sin \Phi')}{(1 + \sin \Phi')} - - - - - - - Eq.(1)$$

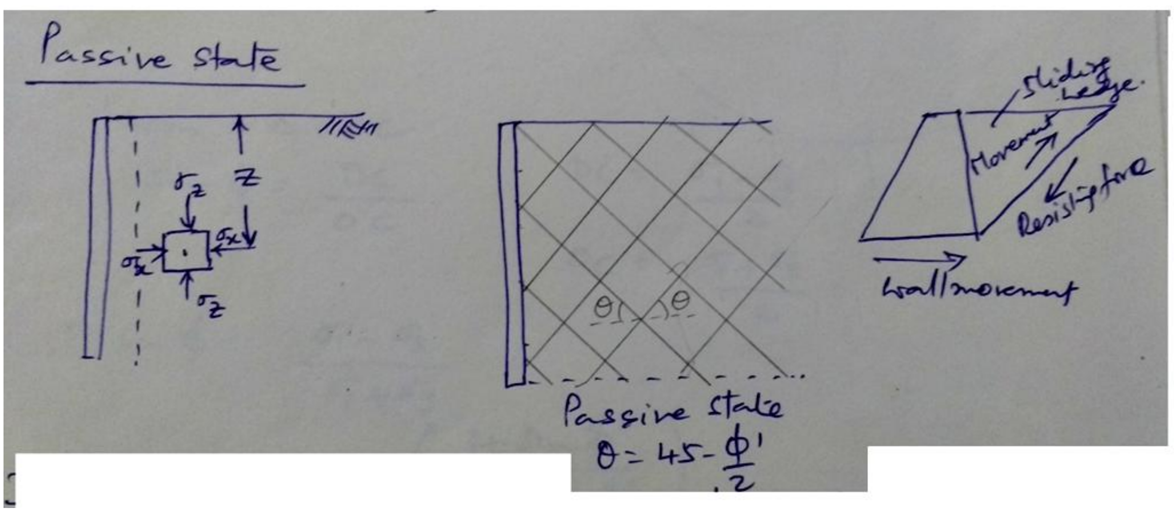
Here $\sigma_1 = \sigma_z$ = weight of soil at depth z
or $\sigma_1 = \gamma z$ (as the soil is assumed to be dry, effective stress = total stress).
The minimum value of $\sigma_x$ is defined as the active earth pressure $p_a$
$\therefore$ $\sigma_1 = \sigma_{x\,min} = p_a$
Substituting these values for $\sigma_1 = \sigma_3$, eqn.(1) becomes
$p_a = \gamma z \left(\frac{1-\sin \Phi'}{1+\sin \Phi'}\right)$

or $p_a = k_a \gamma z$

where $k_a = \frac{1-\sin \Phi'}{1+\sin \Phi'} = \tan^2(45 - \frac{\Phi'}{2})$
$k_a$ = coefficient of earth pressure

when the soil mass is in active Rankine state , two sets of failure planes develop, each inclined at angle θ, equal to $(45 - \frac{\Phi'}{2})$ to the horizontal (which is the direction of major principal plane in the active state).

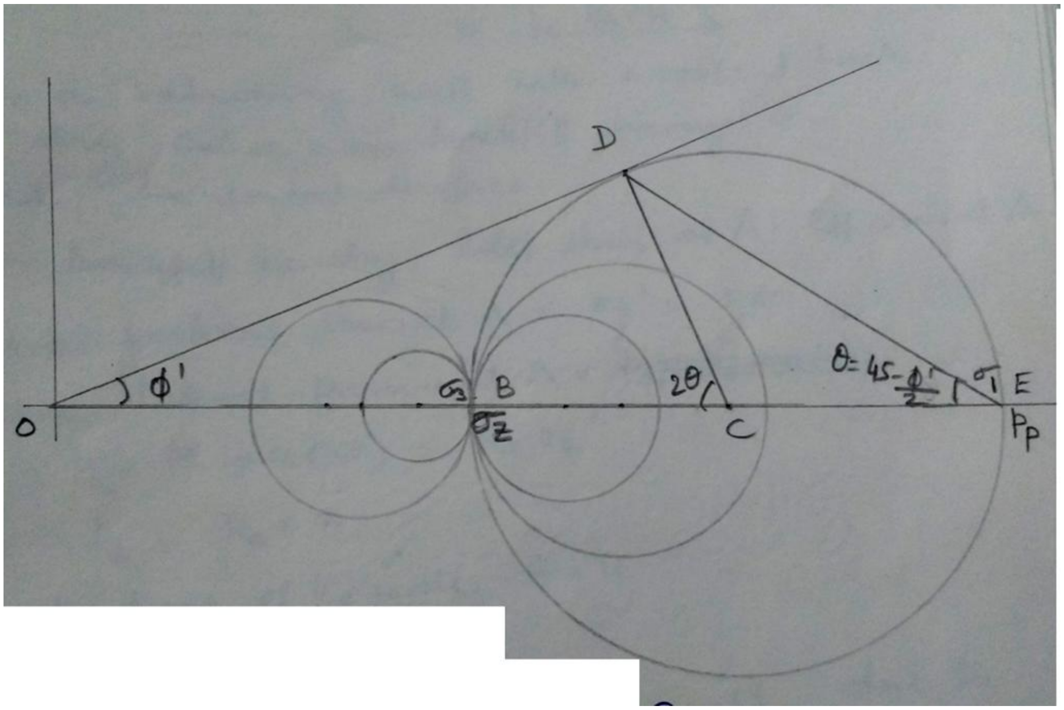
**PASSIVE STATE**




In the passive, the wall moves towards the backfill. There is uniform compression in the horizontal direction. This leads to an increase in the value of $\sigma_x$ from its original value, while $\sigma_z$ remains constant.

As the deformation increases, the diameter of Mohr's stress circle becomes smaller, then reduces to zero ($\sigma_x = \sigma_z$) and subsequently the horizontal stress becomes greater than vertical stress ($\sigma_x > \sigma_z$) and the diameter of the stress circle increases until a state of plastic equilibrium is reached.

For this state, $\sigma_x$ becomes a maximum value and will thus constitute the major principal stress, $\sigma_3$. The maximum value of $\sigma_1$ is reached when the Mohr's circle, drawn with $\sigma_z = \sigma_3$, touches the failure envelope. The soil is then said to be in passive state.




From $\Delta^{le}$ ODC,

$$\sin \Phi = \frac{DC}{Oc}$$

$\therefore \sin \Phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$

or $\sigma_1 = \sigma_3 \left(\frac{1+\sin \Phi'}{1-\sin \Phi'}\right)$

Here, $\sigma 3 = \sigma z = \gamma z$ = weight of soil at depth z

$\sigma_1 = p_p$

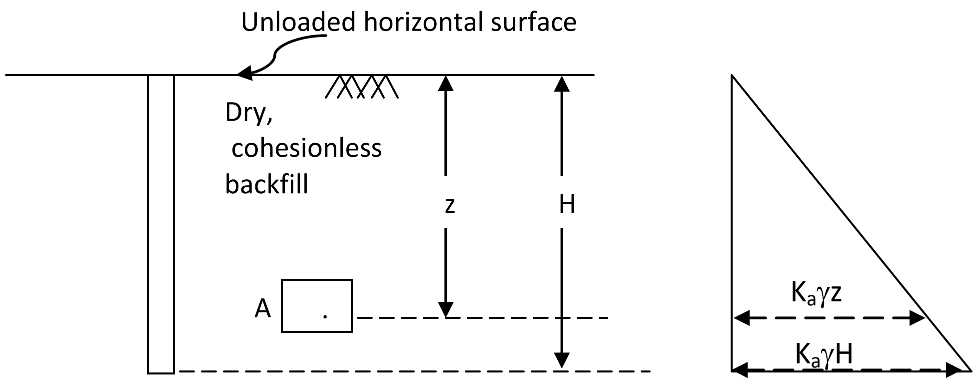
$\therefore p_p = \gamma z \left(\frac{1+\sin \Phi'}{1-\sin \Phi'}\right)$ or $\boxed{p_p = k_p \gamma z}$

Where,

$K_p = \left(\frac{1+\sin \Phi'}{1-\sin \Phi'}\right) = \tan^2\left(45 + \frac{\Phi}{2}\right) = coefficient\ of\ paasive\ earth\ pressure$

**Rankine's Theory**
**Active earth pressure – cohesionless Backfill**

**(a) Dry backfill with no surcharge**



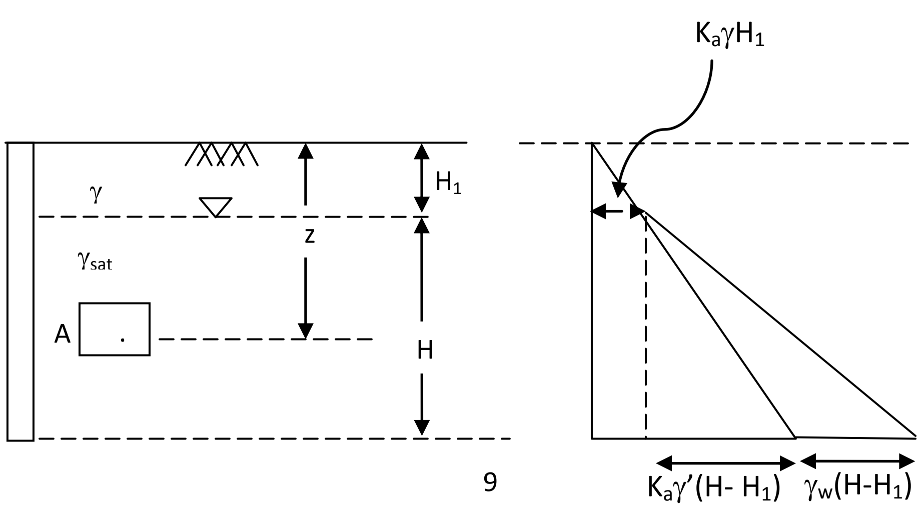

Consider a retaining wall with a vertical back with a dry cohesionless backfill having an unloaded horizontal surface.
As the backfill is dry, total stress at A = Effective stress at A
Effective vertical stress at A = $\sigma'_z = \gamma z$
Effective lateral pressure at A = $p_h = p_0 + u\,(=0) = K_a \sigma'_z$
$\therefore p_h = K_a \gamma z$
At the base of the wall, z = H
$\therefore p_h = K_a \gamma H$

The force per unit length of wall due to active pressure distribution is the total active thrust, $p_A$
Total Active thrust, $P_A$ is the given by the area of the pressure distribution diagram and acts through the centroid of the area

$\therefore P_A = \frac{1}{2}[K_A \gamma H \; x \; H] = \frac{1}{2} K_A \gamma H^2$
$P_A$ acts at a distance $\frac{H}{3}$ above base of the wall.

**(b) Partially Submerged Backfill**

Consider a retaining wall with a vertical back with cohesionless backfill with W.T at a depth $H_1$ from the top.



For the soil above WT, bulk unit weight $\gamma$ is used and for that below the WT, $\gamma_{sat}$ is used.

Consider an element of soil at depth z from top.
Total vertical stress at depth z , $\sigma_z = \gamma H_1 + \gamma_{sat}(z - H_1)$
Pore water pressure at A, $u = \gamma_w(z - H_1)$
$\therefore$ Effective vertical stress at depth z, $\sigma_z' = \gamma H_1 + \gamma_{sat}(z - H_1) - \gamma_w(z - H_1)$

Coefficient of active earth pressure $K_a$ is the ratio of Effective lateral stress to Effective vertical stress

$\therefore K_a = \frac{\sigma_x'}{\sigma_z'}$

$p_0 = \sigma_x' = K_a \sigma_z'$

$= K_a[\gamma H_1 + \gamma_{sat}(z - H_1) - \gamma_w(z - H_1)]$

$= K_a \gamma H_1 + K_a \gamma_{sat}(z - H_1) - K_a \gamma_w(z - H_1)$

The total lateral earth pressure, $p_h = p_0 + u$

$\therefore p_h = K_a \gamma H_1 + K_a \gamma_{sat}(z - H_1) - K_a \gamma_w(z - H_1) + \gamma_w(z - H_1)$

$= K_a \gamma H_1 + K_a(z - H_1)(\gamma_{sat} - \gamma_w) + \gamma_w(z - H_1)$

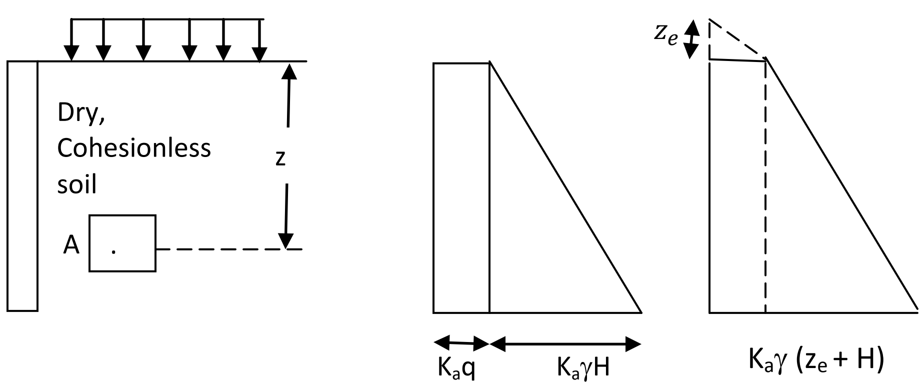
$$p_h = K_a\gamma H_1 + K_a\gamma' (z - H_1) + \gamma_w(z - H_1)$$

At the base of wall, z = H

$$\therefore p_h = K_a\gamma H_1 + K_a\gamma' (H - H_1) + \gamma_w(H - H_1)$$

**(c) Dry backfill with uniform surcharge**

$z_e = \frac{q}{\gamma}$




Effective vertical stress at depth z

$$\sigma_z' = \gamma z + q$$

$$\therefore p_0 = K_a \sigma_z' = K_a[\gamma z + q]$$

As the backfill is dry, $p_0 = p_h$

$\therefore$ At depth z, the total lateral earth pressure $p_h$ is

$$\boxed{p_h = K_a \gamma z + K_a q}$$

At the base of the wall, z = H

$$\boxed{p_h = K_a \gamma H + K_a q}$$

**Note:** The lateral pressure increment due to surcharge is the same at every point of the back of the wall and does not change with depth z.

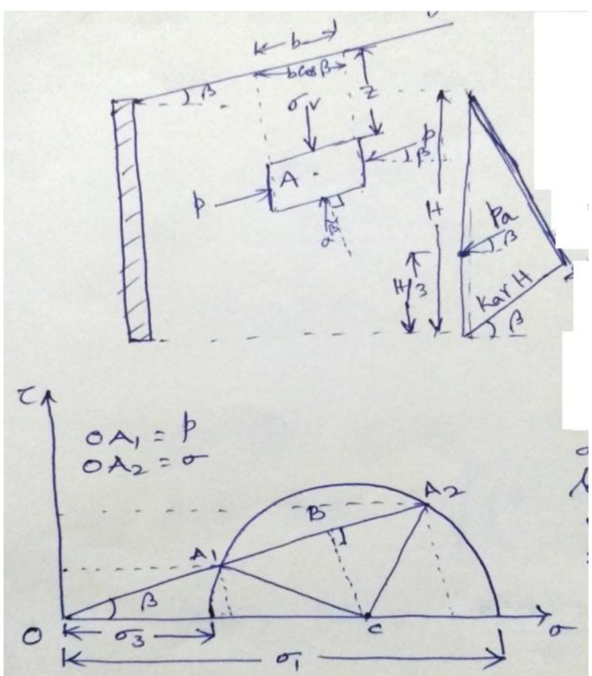
The height of fill, $z_e$, equivalent to uniform surcharge intensity is given by

$$K_a \gamma z_e = K_a q$$

$\therefore z_e = \frac{q}{\gamma}$

Thus, the effect of surcharge of intensity q is the same as that of a fill of height $z_e$ above the ground level.

**Backfill with sloping surface**




Let the sloping surface behind the wall be inclined at angle β. β is call the surcharge angle.

For finding the active earth pressure, an additional assumption that the vertical and lateral stresses are "conjugate" is mode.

**Conjugate planes & Conjugate stresses**

If the stress on a given plane at a given point is parallel to another plane, the stress on the latter plane at the same point must be parallel to the first plane. Such planes are called **conjugate planes** and the stresses acting on them are called **conjugate stresses**.

Consider an element at point A at depth z within the sloping backfill. The top plane of the element is parallel to the ground surface and the other plane conjugate to this plane is vertical.

Let σ and p be the conjugate stresses. Being conjugate, both σ and p have the same angle of obliquity, β.

σ and p are resultant stresses on the two conjugate planes, and not principal stresses.

Let $\sigma_1$ and $\sigma_3$ be the principal stresses. For stability requirement in a cohesionless soil,

$$\sin \Phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} --------- Eq.(1)$$

Consider the Mohr's circle corresponds to the principal stress intensities, $\sigma_1$ and $\sigma_3$.

The obliquity of σ and p is β. Hence through O, draw a line at obliquity β, to cut the Mohr circle at $A_1$ and $A_2$. Thus $OA_1$ represents the resultant stress p and $OA_2$ represents the resultant stress σ.

Draw OB perpendicular $A_1A_2$.

$$OB = OC \cos\beta, \qquad OC = \frac{\sigma_1+\sigma_3}{2}$$

$$\therefore OB = \frac{\sigma_1+\sigma_3}{2} \cos\beta --------- Eq.(2)$$

$$BC = OC \sin\beta$$

$$\therefore BC = \frac{\sigma_1+\sigma_3}{2} \sin\beta ----------- Eq.(3)$$

$$A_1B = BA_2 = \sqrt{A_1C^2 - BC^2} = \sqrt{\left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 - \left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \sin^2\beta}$$

From Eq.(1), $\sigma_1 - \sigma_3 = (\sigma_1 + \sigma_3)\sin\Phi$

$\therefore$

$$A_1B = BA_2 = \sqrt{A_1C^2 - BC^2} = \sqrt{\left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \sin^2\Phi - \left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \sin^2\beta}$$

$$A_1B = BA_2 = \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\Phi - \sin^2\beta}$$

$$\sigma = OA_2 = OB + BA_2 = \frac{\sigma_1 + \sigma_3}{2}\cos\beta + \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\Phi - \sin^2\beta}$$

or $\quad \sigma = \frac{\sigma_1 + \sigma_3}{2}\left[\cos\beta + \sqrt{\sin^2\Phi - \sin^2\beta}\right]$

$$p = OA_1 = OB - A_1B = \frac{\sigma_1 + \sigma_3}{2}\cos\beta - \frac{\sigma_1 + \sigma_3}{2}\sqrt{\sin^2\Phi - \sin^2\beta}$$

or

$$p = \frac{\sigma_1 + \sigma_3}{2}\left[\cos\beta - \sqrt{\sin^2\Phi - \sin^2\beta}\right]$$

$$K = \frac{p}{\sigma} = \frac{\cos\beta - \sqrt{\sin^2\Phi - \sin^2\beta}}{\cos\beta + \sqrt{\sin^2\Phi - \sin^2\beta}}$$

K is called as conjugate ratio or Rankine's lateral pressure ratio.
For the pressure case,

$$\sigma = \frac{\gamma z\, b\cos\beta}{b} = \gamma z\ \cos\beta$$

p = lateral earth pressure, $p_a$.

$$p_a = \gamma z\ \cos\beta\ .\frac{\cos\beta - \sqrt{\sin^2\Phi - \sin^2\beta}}{\cos\beta + \sqrt{\sin^2\Phi - \sin^2\beta}}$$

$$p_a = K_a\gamma z$$

Where

$$K_a = \cos\beta\ .\frac{\cos\beta - \sqrt{\sin^2\Phi - \sin^2\beta}}{\cos\beta + \sqrt{\sin^2\Phi - \sin^2\beta}}$$

For β = 0, i.e, horizontal ground surface,

$$K_a = \frac{1 - \sin\Phi}{1 + \sin\Phi}$$

The total active pressure $P_a$ per unit length for the wall of height H is

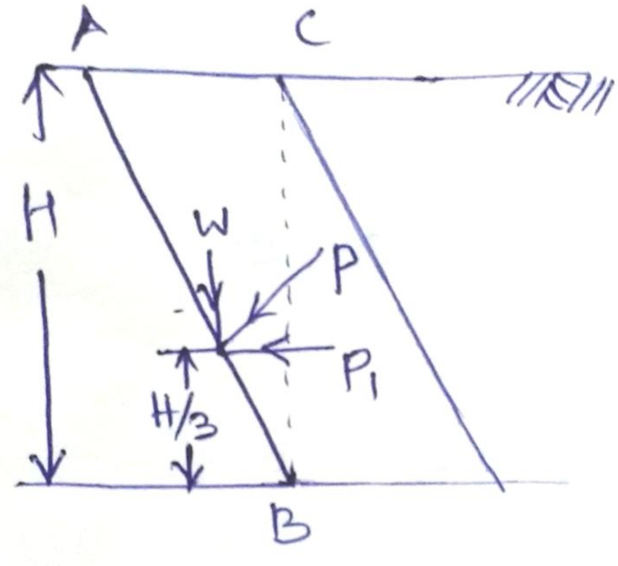
$$P_a = \frac{1}{2}K_a\gamma H^2$$

Acting at $\frac{H}{3}$ above the base of the wall in direction parallel to the ground surface.

If the backfill is submerged, the lateral pressure due to submerged weight of soil will act at β to horizontal, while lateral pressure due to water will act normal to the wall.

**Inclined Back and Surcharge**

Consider a retaining wall with an inclined back corresponding a backface with horizontal surface.



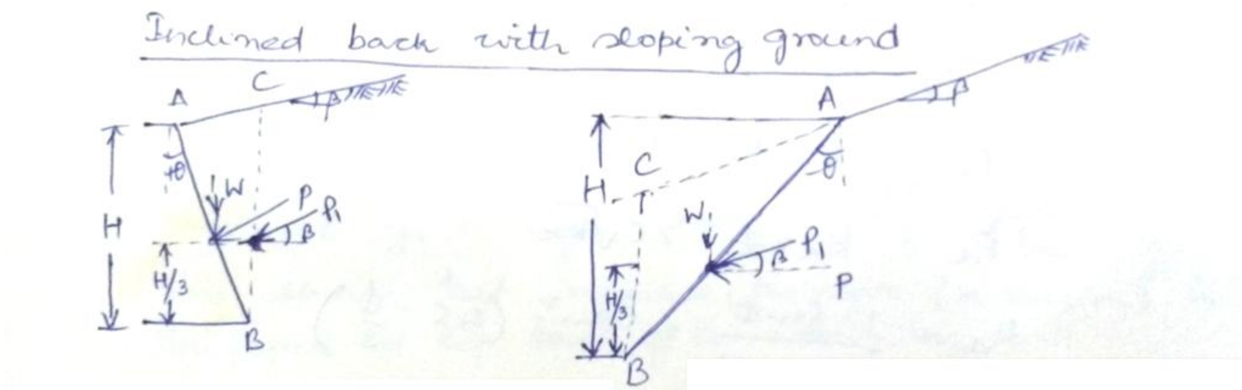

The total active pressure $P_1$ is first calculated on a vertical plane BC passing through the heel B.

The total pressure P is the resultant of the horizontal pressure $P_1$ and the weight W of the wedge ABC

$\therefore P = \sqrt{W^2 + {P_1}^2}$

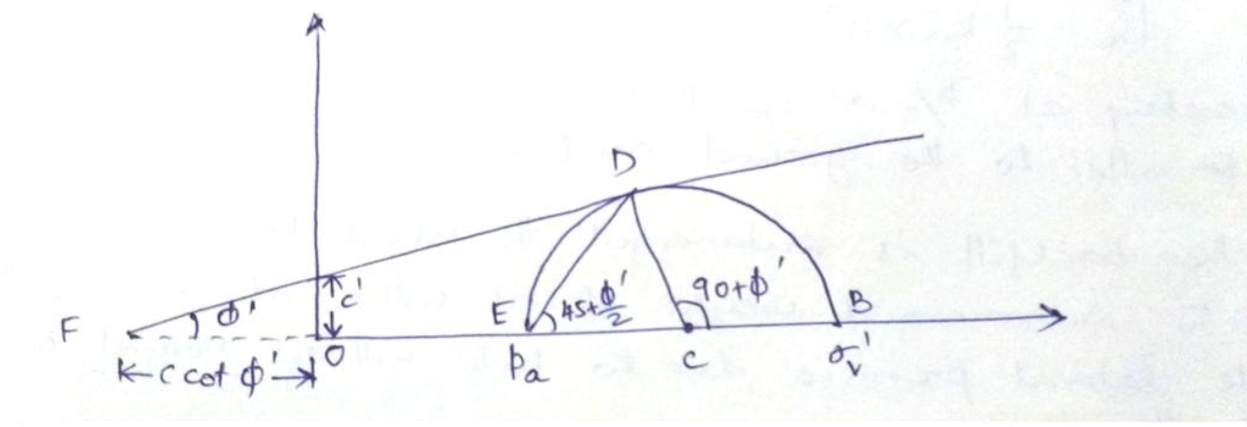
Where, $P_1 = \frac{1}{2} K_a \gamma H^2$

**Inclined back with sloping ground**




The active earth pressure is first calculated on a vertical plane passing through the heel and intersecting the surface of the backface or its extension point at C. The height of vertical plane is represented by BC.

The resultant of P is the vector sum of $P_1$ and $W_1$, where W is the weight of soil contained in the wedge ABC.




From Δ FCD, $\sin\Phi' = \frac{CD}{FC} = \frac{CD}{FO + OC}$

or

$$\sin\Phi' = \frac{(\sigma_1 - \sigma_3)/2}{c\ \cot\Phi' + (\sigma_1 + \sigma_3)/2}$$

or $\frac{\sigma_1 - \sigma_3}{2} = \left(\frac{\sigma_1 + \sigma_3}{2}\right)\sin\Phi' + c\ \cos\Phi'$

or $\frac{\sigma_1}{2}(1 - \sin\Phi') = \frac{\sigma_3}{2}(1 + \sin\Phi') + c\ \cos\Phi'$

or $\sigma_3 = \left(\frac{1 - \sin\Phi'}{1 + \sin\Phi'}\right)\sigma_1 - 2c'\ \frac{\cos\Phi'}{1 + \sin\Phi'}$

or $\sigma_3 = \sigma_1\tan^2\left(45 - \frac{\Phi'}{2}\right) - 2c'\ \tan\left(45 - \frac{\Phi'}{2}\right)$

or $\sigma_3 = active\ earth\ pressure$

$\sigma_1 = \gamma z$

$$P_a = \gamma z\tan^2\left(45 - \frac{\Phi'}{2}\right) - 2c'\ \tan\left(45 - \frac{\Phi'}{2}\right)$$

$$P_a = K_a\gamma z - 2c'\sqrt{K_a}$$

$$K_a = \frac{1 - \sin\Phi'}{1 + \sin\Phi'} = \tan^2\left(45 - \frac{\Phi'}{2}\right)$$

$$\frac{\cos\Phi'}{1 + \sin\Phi'} = \tan\left(45 - \frac{\Phi'}{2}\right) = \sqrt{K_a}$$

**Rankine's Theory**
**Active Earth Pressure – Cohesive Soils**

The principal stress relationship on a failure plane is given by

$$\sigma_1 = \sigma_3\tan^2\alpha + 2c\ \tan\alpha \quad ------eq.(1)$$

Consider a retaining wall with a smooth, vertical back retaining a horizontal backface.

At any depth z,

$$\sigma_1 = \gamma z \quad and \quad \sigma_3 = lateral\ pressure,\ p_a$$

Substituting in Eq.(1)

$\gamma z = \; p_a \tan^2 \alpha \; + 2c \; \tan \alpha$

$$K_a = \cot^2 \alpha = \cot^2 \left(45 + \frac{\Phi'}{2}\right) = \frac{1 - \sin \Phi'}{1 + \sin \Phi'}$$

or $\quad p_a \tan^2 \alpha = \gamma z \; - 2c \; \tan \alpha$

or $\quad p_a = \gamma z \; \cot^2 \alpha - 2c \; \cot \alpha \quad ------Eq.(2)$

$$p_a = \gamma z \, K_a \; - 2c \sqrt{K_a}$$

Where $\alpha = 45 + \frac{\Phi}{2}$

Eq.(2) is known as Bell's equation for the lateral pressure of cohesive soils.

$$K_a = \cot^2 \alpha = \cot^2 \left(45 + \frac{\Phi'}{2}\right) = \frac{1 - \sin \Phi'}{1 + \sin \Phi'}$$

At z = 0, $\quad p_a = \; -2c \; \cot \alpha$

When $p_a = 0$,

$$\gamma z \; \cot^2 \alpha \; = 2c \; \cot \alpha$$

$$\gamma z \; \cot \alpha = 2c$$

$$z = z_0 = \frac{2c}{\gamma} \tan \alpha = \frac{2c}{\gamma} \frac{1}{\sqrt{K_a}}$$

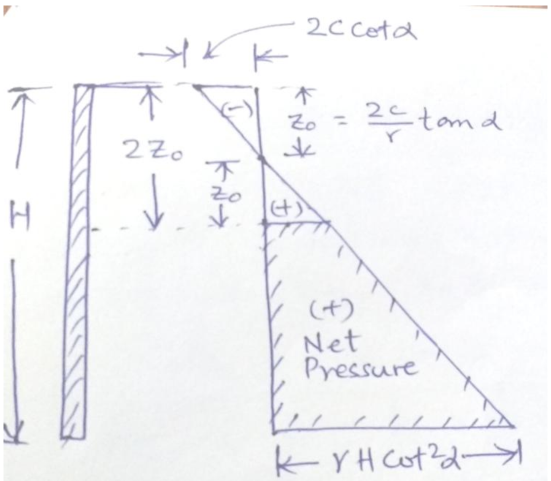
This shows that negative pressure (or tension) is developed at top level of the retaining wall.

The tension decreases to zero at a depth $z_0 = \frac{2c}{\gamma} \tan \alpha$

For $z > z_0$, the pressure $p_a$ is positive.

At z = H, $p_a = \gamma H \; \cot^2 \alpha - 2c \; \cot \alpha$

Because of negative pressure (or tension), a tension crack is usually developed in the soil near the top of the wall, upto a depth $z_0$.



Pressure at the base of the wall is given by

$$p_a = \gamma H \cot^2 \alpha - 2c \cot \alpha$$

For c = 0, the pressure $p_a$ would have been

$$p_a = \gamma H \cot^2 \alpha$$

∴ The effect of cohesion is to reduce the pressure intensity everywhere by $2c \cot \alpha$.

The total net pressure is

$$P_a = \int_0^H p_a . dz = \int_0^H (\gamma z \cot^2 \alpha - 2c \cot \alpha)\, dz$$

Before formation of the crack →

$$P_a = \frac{1}{2} \gamma H^2 \cot^2 \alpha - 2cH \cot \alpha$$

← $P_a$ considering negative pressure

The net pressure up to a depth $2z_0$ is zero. This means cohesive soil should be able to stand unsupported with a vertical face up to a depth $2z_0$

∴ The critical height of an unsupported (no lateral support) vertical cut in a cohesive soil is

$$H_c = 2\, z_0 = \frac{4\, c}{\gamma} \tan \alpha$$

**Effect of tension crack on Pa**

As cracks do occur and the soil does not necessarily remain adhered to the top portion of the wall, up to a height $z_0$, it is …. To neglect the negative pressure and consider whole of the positive pressure below $z_0$.

The total lateral thrust on the wall is

$$P_a = \int_{z_0}^H (\gamma z \cot^2 \alpha - 2c \cot \alpha)\, dz$$

$$= \frac{1}{2} \gamma (H^2 - z_0^2) \cot^2 \alpha - 2c\, (H - z_0) \cot \alpha$$

Substituting $z_0 = \frac{2c}{\gamma} \tan \alpha$

After formation of crack

$$P_a = \frac{1}{2} \gamma H^2 \cot^2 \alpha - 2c\, H \cot \alpha + \frac{2c^2}{\gamma}$$

← Pressure neglecting negative pressure

**<u>Backfill with surcharge</u>**

If the backfill carries a surcharge of uniform intensity q per unit area, the lateral pressure is increased by $K_a q \;\; (or)\; q \cot^2 \alpha$ everywhere.

$$\therefore p_a = \gamma z \cot^2 \alpha - 2c \cot \alpha + q \cot^2 \alpha$$

At z = 0,

$$p_a = q\cot^2\alpha - 2c\cot\alpha$$

The depth $z_0$ at which $p_a = 0$ is

$$0 = \gamma z_0\cot^2\alpha - 2c\cot\alpha + q\cot^2\alpha$$

$$\boxed{z_0 = \frac{2c}{\gamma}\tan\alpha - \frac{q}{\gamma}}$$

**Submerged backfill**

If the water table exists at a depth $H_1$ below the top of the wall, lateral pressure at depth z ( $>H_1$) is

$$p_a = [\gamma H_1 + \gamma'(z - H_1)]\cot^2\alpha - 2c\cot\alpha + \gamma_w(z - H_1)$$

**Backfill of inclined saturated clay**

For temporary works or immediately after construction of retaining wall, $\Phi = \Phi_u = 0$ is assumed

$$\therefore K_a = \frac{1-\sin\Phi}{1+\sin\Phi} = 1$$

$$\therefore p_a = \gamma_{sat}\,z - 2c_u$$

$$P_a = \frac{1}{2}\gamma_{sat}(H^2 - z_0^2) - 2c_u\,(H - z_0)$$

$$z_0 = \frac{2c_u}{\gamma_{sat}}$$

**Rankine's theory – Passive Earth pressure**

**<u>Cohesionless backfill – No Surcharge</u>**

In case of passive state of plastic equilibrium, the lateral pressure is the major principal stress while the vertical pressure is the minor principal stress.

$$\therefore \sigma_h = p_p = \sigma_1$$

$$\sigma_v = \sigma_3 = \gamma z$$

Substituting in the principal stress relationship,

$$\sigma_1 = \sigma_3\tan^2\alpha$$

$$p_p = \gamma z\tan^2\alpha$$

$$p_p = k_p\,\gamma z$$

$$k_p = \tan^2\alpha = \frac{1+\sin\Phi}{1-\sin\Phi}$$

The distribution of passive earth pressure is triangular, with max. value of $k_p\,\gamma H$ at the base.

$$P_p = \frac{1}{2}k_p\,\gamma H^2$$

**With surcharge**

$p_p = k_p\ (\gamma z + q)$

**Top surface inclined at surcharge angle β**

$$p_p = \gamma z \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\Phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\Phi}}$$

$$p_p = k_p\ \gamma z$$

**Cohesive backfill**

The principal stress relationship is

$$\sigma_1 = \sigma_3\ \tan^2\alpha + 2c\ \tan\alpha$$

For passive pressure

$$\sigma_1 = \sigma_h = p_p\ \ ;\ \sigma_3 = \sigma_v = \gamma z$$

$$\therefore p_p = \gamma z \tan^2\alpha + 2c\ \tan\alpha$$

At z = 0, $p_p = 2c\ \tan\alpha$ ;

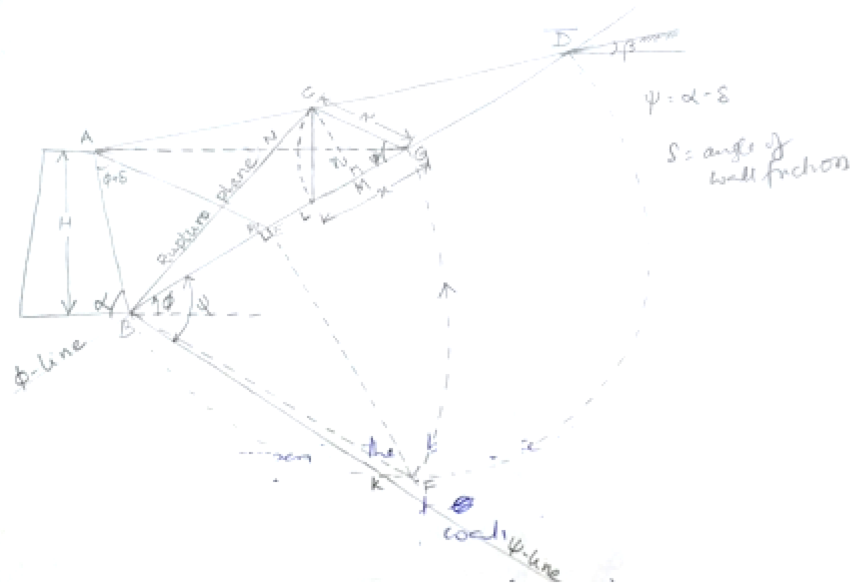
At z = H, $p_p = \gamma H \tan^2\alpha + 2c\ \tan\alpha$

Total Pressure,

$$P_p = \int_0^H p_p\, dz$$

$$P_p = \frac{1}{2}\gamma H^2\ \tan^2\alpha + 2c\ H\ \tan\alpha$$

**Rebhan's Graphical method (Active case)**



1. Let AB represent the backface of the wall and AD the backfill surface.
2. Draw BD inclined at Φ with the horizontal from the heel B of the wall to meet the backfill surface in D.
3. Draw BK inclined at Ψ (= α - δ ) with BD, which is the Ψ line.
4. Through A, draw AE parallel to Ψ line to meet BD in E.
5. Describe a semi - circle on BD as diameter.
6. Erect a perpendicular to BD at E to meet the semi circle in F.
7. With B as center and BF as radius draw an arc to meet BD in G.
8. Through G, draw a parallel to the Ψ - line to AD in C.
9. With G as center and GC as radius draw an arc to cut BD in L. Join CL and also draw a perpendicular CM from C on to LG.

BC is the required rupture surface.
Since area of Δ ABC = area of Δ BCG and BC is their common tangent, their altitude on BC must be equal.
or $AN\ \sin\angle ANB = NG\ \sin\angle GNC$
or AN = NG $\quad (\because\ \angle ANB = \angle GNC - opp.angles)$

Thus if AN and NG are measured and found equal, the construction is correct.
The active thrust $P_a$ is
$P_a = \frac{1}{2}\gamma x^2 \sin\Psi,$ where CG = LG = x
$= \gamma.(area\ of\ \Delta\, CGL)$
$= \frac{1}{2}\gamma.\text{x}.\text{n}$ where n = CM

**Validity of the method**

$$c^2 = BG^2 = BF^2 = BE^2 + EF^2$$

But $EF^2 = BE.ED$ from the properties of a circle

$\therefore c^2 = BE^2 + BE.ED$

$\quad = BE\,(BE + ED)$

$c^2 = b\,.d$

This is Poncelet's rule which implies Rebhann's condition automatically; hence the validity of the construction.

**Special cases**

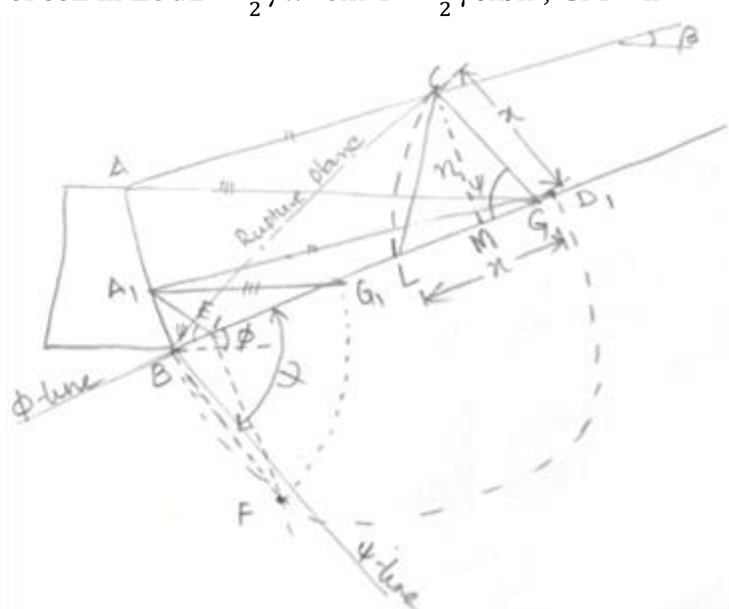
**1. β nearly equal to Φ**

When β is nearly equal to Φ , the Φ-line and backfill surface meet at a very large distance which may not accommodated on the drawing. The graphical procedure for this case is as follows:

1. Represent the backface of the wall AB and backfill surface through A.
2. Draw the Φ line through B inclined at Φ with the horizontal.
3. Draw the Ψ-line BK through B inclined at Ψ with Φ-line.
4. Choose a convenient point $D_1$, on the Φ-line, draw the semi-circle on $BD_1$ as the diameter.
5. Draw $D_1A_1$ parallel to the backfill surface to meet the wall in $A_1$.
6. Through $A_1$ draw $A_1E_1$ parallel to the Ψ-line to meet Φ-line in $E_1$.
7. Erect a perpendicular $E_1F_1$ to the Φ-line at $E_1$ to meet the semi circle in $F_1$.
8. With B as center and $B_1F_1$ as radius, draw an arc $F_1G_1$ to meet Φ-line in $G_1$.
9. Through A draw AG parallel to $A_1G_1$ to meet the Φ-line in G.
10. Through G draw a line parallel to the Ψ-line to meet the backfill surface in C.
11. With G as center and GC as radius, draw an arc to meet Φ-line in L.
12. Join CL and drop CM perpendicular GL.

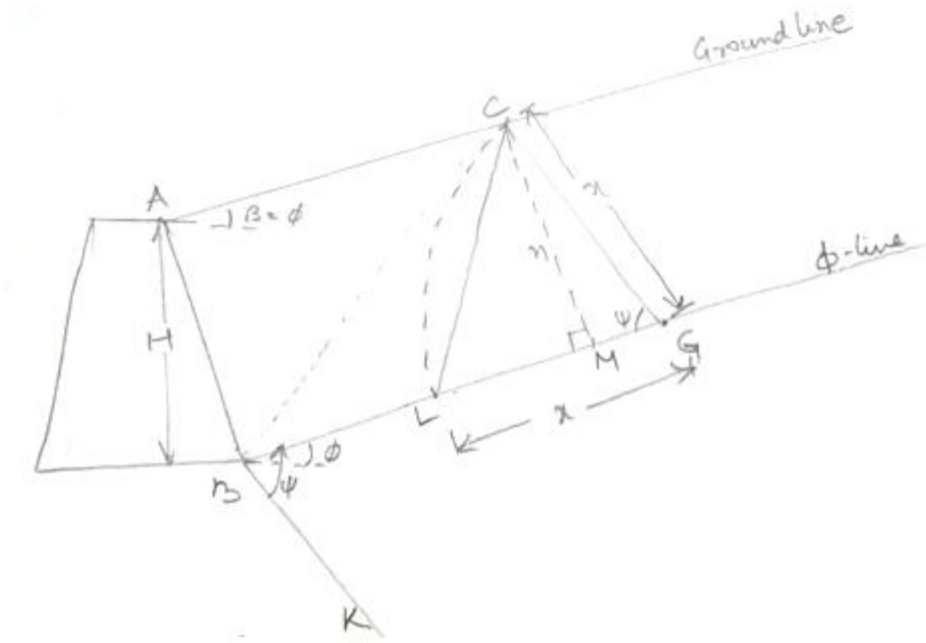
    BC is the required rupture surface.

    $P_a$ = weight of soil in $\Delta CGL = \frac{1}{2}\gamma x^2 \sin \Psi = \frac{1}{2}\gamma\,.\,x.\,n$ ; CM = n



Special case

**2. Φ = β**




When β exactly equal to Φ, ground line and Φ-line are parallel and will meet only at infinity. The points C and D, and the triangle CGL exist at infinity. However, the triangle CGL can be constructed anywhere between the Φ-line and the ground line.

1. Draw the ground line and the Φ-line.
2. Draw the Ψ-line BK through B at an angle Ψ with Φ-line.
3. From any convenient point G on the Φ-line, draw a line parallel to Ψ-line to meet the ground line in C.
4. With G as center and GC as radius, draw an arc to cut the Φ-line in L.
5. Join CL and drop CM perpendicular to LG

$$P_a = \gamma(\Delta\, CGL) = \frac{1}{2}\gamma x^2 \sin \Psi = \frac{1}{2}\gamma \,.\, \mathrm{x.\,n}$$

**Culmann's Graphical Method – Active Earth Pressure.**

This method may be conveniently used for ground surface of any shape, for different types of surcharge loads, and for layered backfill with different unit weights for different layers.

Fig…

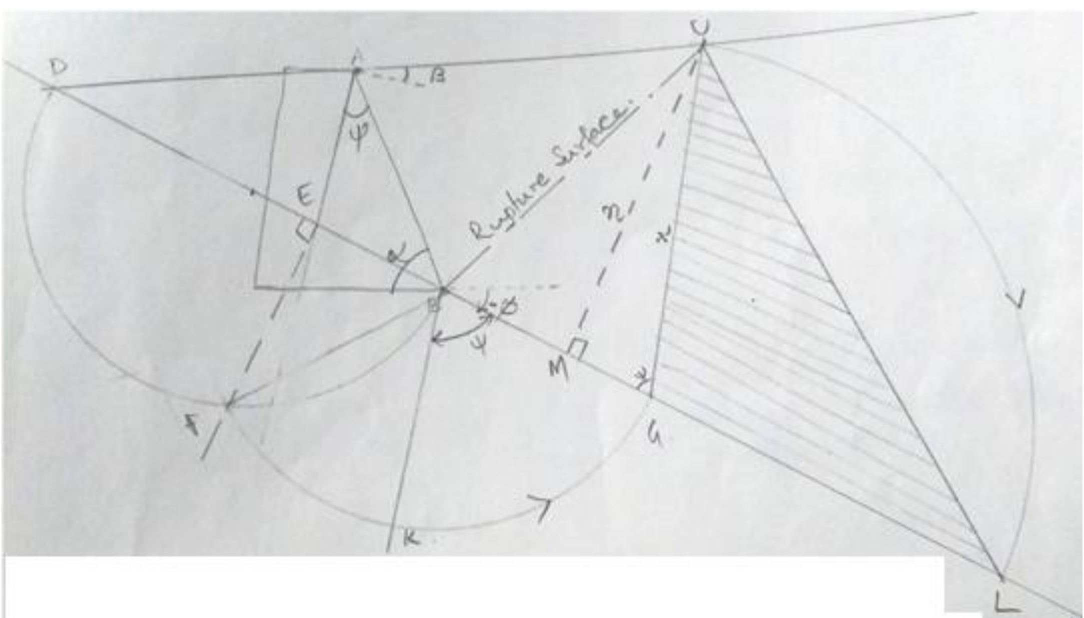
1. Draw the ground line , Φ-line, Ψ-line and back of wall
2. Choose an arbitrary failure plane $BC_1$. Calculate the weight of the wedge $ABC_1$ and plot it as B-1 to a conventional scale (Force Scale) on the Φ-line.
3. Draw 1-1′ parallel to the Ψ-line through 1 to meet $BC_1$ in 1′. 1′ is the point on the Culmann line.
4. Similarly, take some more failure planes $BC_2$, $BC_3$… and repeat steps 2 and 3 to establish points 2′ , 3′…..

5. Join B, 1′,2′,3′…..etc smoothly to obtain the Culmann curve.
6. Draw tangent t – t to the Culmann line parallel to the Φ-line. Let the point of tangency be F′.
7. Draw F′F parallel to Ψ-line to meet the Φ-line in F.
8. Join BF′ and produce it to meet the ground line in C.
9. BF′C represents the failure surface and F′F represents $P_a$ to the same scale as that choosen to represent the weights of wedges.

Note: If the upper surface of the backfill is a plane, the weights of wedges will be proportional to the distances $l_1$, $l_2$,….(bases), such they have a common height $H_1$. Thus B-1, B-2, etc may be made equal or proportional to $l_1$, $l_2$, …. Etc.
Compare BF with weight of the wedge ABC.

**Rebhann (Poncelet) construction for the determination of passive resistance.**

In the passive case, the signs of the angles of internal friction of soil and wall friction have to be reversed.




1. Draw the wall face AB and backfill surface through A.
2. Draw the Φ-line through B at Φ below the horizontal. Let the Φ-line produced meet the ground line extended backwards in D.
3. Draw the Ψ-line BK through B at Ψ = α + δ clockwise from Φ-line.
4. Through A, draw AE parallel to Ψ-line to meet BD in E.
5. Describe a semi-circle on BD as diameter.
6. Erect a perpendicular to BD at E to meet the semicircle in F.
7. With B as center and BF as radius draw an arc to meet DB produced in G.
8. Through G, draw a line parallel to the Ψ-line to meet  the backfill surface in C.

9. With G as center and GC as radius draw an arc to cut DB produced beyond G in L. Join CL and also draw perpendicular CM from C on to LG.
   BC is the required rupture surface.

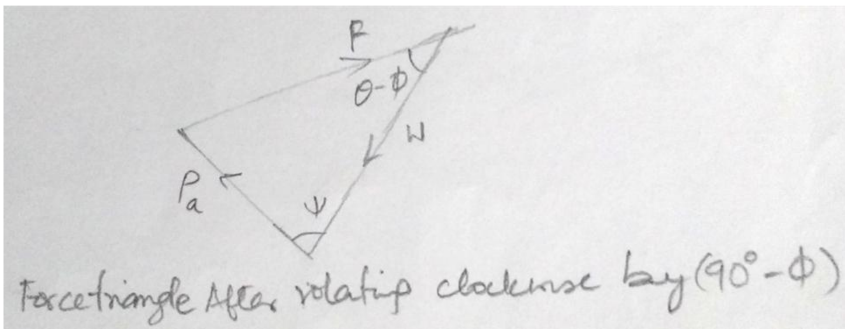
The passive resistance $P_p$ is given by

$P_p = \gamma.(area\ of\ \Delta CGL)$

$= \frac{1}{2}\ \gamma.x^2\ \sin\Psi$

$= \frac{1}{2}\ .\gamma.x\ .n$

Imagine the force triangle to be rotated clockwise through an angle (90° - Φ), as to bring the vector $\vec{W}$ parallel to the Φ-line ; in that case, reaction R will be parallel to the rupture surface and the active thrust $P_a$ parallel to the Ψ-line.




The force triangle and Δ BF′F are similar

$\therefore \frac{P_a}{W} = \frac{F'F}{BF}$

or $P_a = W.\frac{F'F}{BF}$

Where W is the weight of the wedge ABC

$W = \gamma.\frac{1}{2} \text{ x } l \text{ x } H_1$

$\therefore\ P_a = \frac{1}{2}\ \gamma H_1\ l\ .\frac{F'F}{BF}$

But BF = $l$

$$P_a = \frac{1}{2}\ \gamma H_1\ .F'F$$

10. To locate the point of application of the resultant pressure, draw a line parallel to critical slip plane BC, through the centroid of the sliding wedge ABC.