

## UNIT - III

### BEARING CAPACITY OF SHALLOW FOUNDATIONS

#### Introduction

For most structures, the earth provides the ultimate support. The behavior of the supporting ground will therefore affect the stability of the structure. The supporting ground invariably a soil which is weaker than any construction material.

Structural foundations are the sub-structure elements which transmit the structural load to the earth in such a way that the supporting soil is not over stressed and not undergo deformations that would cause excessive settlements of the structure.

The various types of structural foundations can be broadly grouped into two categories.

- (i) Shallow foundations
- (ii) Deep foundations

#### Shallow foundations

A shallow foundation transmits structural load to the soil strata at a relatively shallow depth.

Terzaghi's definition of a shallow foundation is one which is laid at a depth  $D_f$  not exceeding the width  $B$  of the foundation i.e.,  $\frac{D_f}{B} \leq 1$ .

Shallow foundations are constructed in open excavations and the disturbance of soil is minimal. For reasons of economy, shallow foundations are preferred to deep foundations wherever possible.

The various types of shallow foundations are

- (i) **Strip footing or continuous footing:** In strip footing, the length of the footing is much greater than its width ( $L >> B$ ), commonly used below walls.
- (ii) **Spread footing, square or circular in section,** is commonly used below a column (isolated) or below more than one column (considered). The shape of spread footing is commonly rectangular or trapezoidal in plan.
- (iii) **Raft or Mat foundation:** Covers the entire area of a structure, transmitting entire structural load from several columns.
- (iv) **Moderately deep and Deep foundations**

Foundations with  $D_f / B$  ratio greater than 1 but less than 15 are moderately deep.

Deep foundations have  $\frac{D_f}{B}$  ratio greater than 15.

In deep foundation load is supported partly by frictional resistance around the foundation and the rest by bearing at the base of the foundation.

The method of construction of deep foundation makes it impossible to visually inspect the construction. The disturbance of soil extends to a larger zone all along the length of a deep foundation.

The choice of a particular type of foundation depends on

- (i) The magnitude of loads
- (ii) The nature of subsoil strata
- (iii) The nature of super structure and its specific requirements.

### **General requirements of a foundation**

For a satisfactory performance, a foundation must satisfy the following basic criteria:

- (i) Location and depth criterion
- (ii) Shear failure criterion or bearing capacity criterion
- (iii) Settlement criterion

### **Location and Depth criterion**

A foundation must be properly located and founded at such a depth that its performance is not adversely affected by factors such as lateral expulsion of soil from beneath the foundation. Seasonal volume changes caused by freezing and thawing, swelling and shrinkage and the presence of adjoining structures.

IS: 1904-1986 recommends that a foundation should be located at a minimum depth of 50 cm below the natural ground surface.

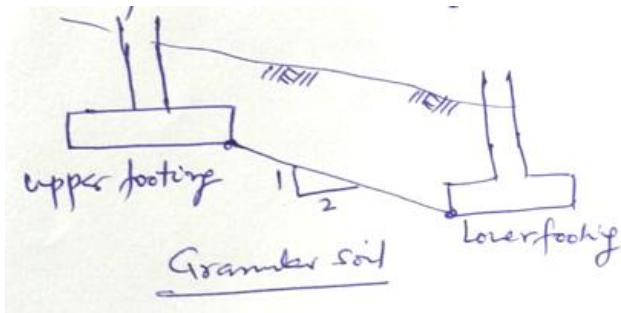
The foundation must be located below the zone of volume change, when volume change is expected. The zone of seasonal variation in water content varies in thickness from 1.5m to 3.5m in black cotton soils.

In case of fine sands and silts, in areas of extremely low temperatures, the foundation must be placed below the zone of frost heave.

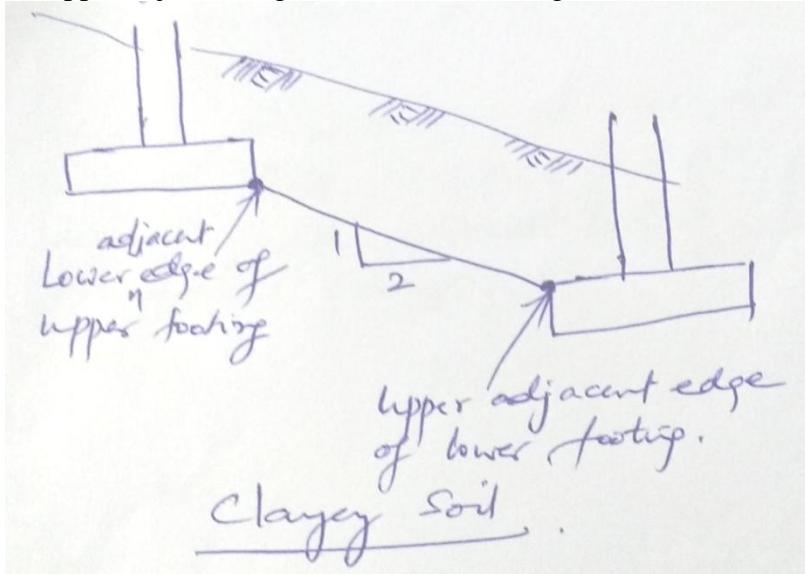
The depth of foundation for any structure in rivers must be sufficiently below the deepest scour level.

### **IS 1904-1986 Recommendations**

1. When the ground surface slopes downward adjacent to a footing, the sloping surface should not encroach upon a frustum of bearing material under the footing having sides which make an angle of  $60^\circ$  with horizontal for rock and  $30^\circ$  for soil. The horizontal distance between lower edge of the footing to the sloping surface shall be at least 60 cm for rock and 90 cm for soil.
2. For footings in granular soils, the line joining the lower edges of adjacent footings should not have a slope steeper than 2H to 1V.

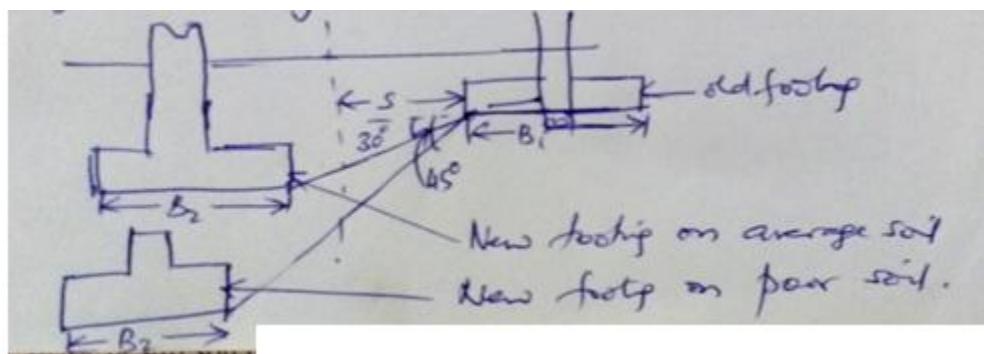


3. In clayey soils, the slope of the line joining the lower adjacent edge of the upper footing and the upper adjacent edge of the lower footing should not be steeper than 2H to 1V.



**4. Foundation for a new structure at an adjacent site.**

The adjacent edge of the new footing must be atleast at a distance 'S' from the edge of the existing footing, where S is the width of the larger footing.



The line from the edge of new footing to the edge of existing footing should make an angle of  $45^\circ$  or less with horizontal plane.

i.e., the distance should be greater than the difference in elevation between adjacent footings. The recommendation remains the same even if positions of new and existing footings are interchanged. This provision ensures that stress overlap due to adjacent footings does not assume sufficient proportions.

When a new footing is placed lower than an old footing, the existing structure may be endangered because of the lateral flow of soil from beneath the existing footing. The excavation must therefore, not be too close to the existing footing.

Presence of underground utilities, cavities, old mine tunnels, soft fill material etc. must be carefully examined before working out the foundation depth.

## DEFINITIONS

For a footing constructed with its base at a depth  $D_f$  below the ground surface, the total pressure  $q_g$  at the base of the footing will be due to the weight of the superstructure, weight of the footing and weight of the soil fill over the footing. This total pressure is known as gross pressure or the gross loading intensity.

However deformation of the soil below the base of the footing is caused only by pressure over and above that which existed before the construction of the footing and the superstructure. The difference between gross pressure and the overburden pressure  $\gamma D_f$  at the base of the footing is called net pressure or net loading intensity.

1. **Ultimate Bearing Capacity:** The maximum gross intensity of loading that the soil can support before it fails in shear is called the ultimate bearing capacity ( $q_u$ ).
2. **Net Ultimate Bearing Capacity ( $q_{nu}$ ):** It is the net increase in pressure at the base of foundation that causes shear failure of soil.

$$q_{nu} = q_u - \gamma D_f$$

3. **Net Safe Bearing Capacity ( $q_{ns}$ ):** It is the net soil pressure which can be safely applied to the soil considering only shear failure.

$$q_{ns} = \frac{q_{nu}}{F} \quad [F = \text{factor of safety is 2.5 to 3 (usually 3)}]$$

4. **Gross Safe Bearing Capacity ( $q_s$ ):** It is the maximum gross pressure which the soil can carry safely without shear.

$$q_s = q_{ns} + \gamma D_f = \frac{q_{nu}}{F} + \gamma D_f$$

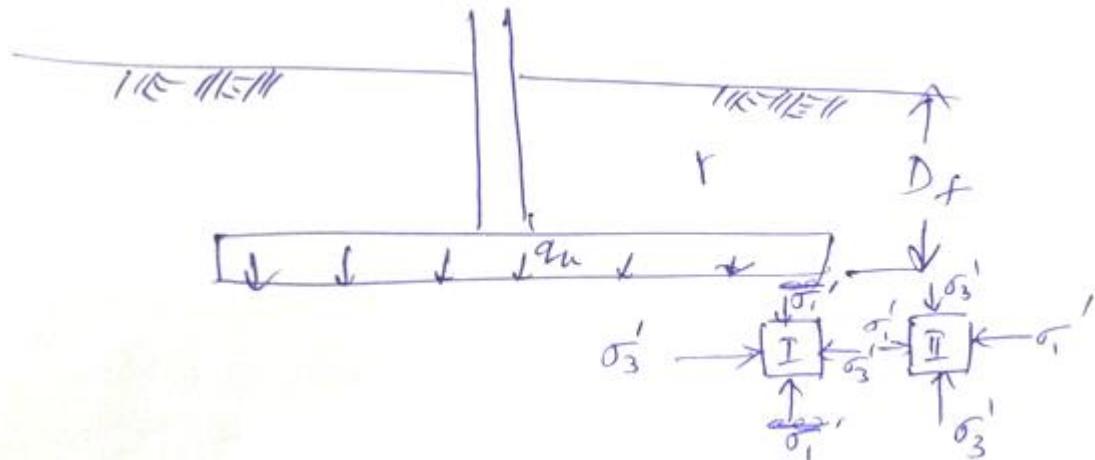
Since the additional pressure due to  $\gamma D_f$  available in full, factor of safety is not used to this term.

5. **Net Safe Settlement Pressure ( $q_{np}$ ):** It is the net pressure which the soil can carry without exceeding the allowable settlement (usually varies between 25 mm to 40 mm for individual footings).
6. **Net Allowable Bearing Pressure ( $q_{na}$ ):** Max. net intensity of loading that can be imposed on the soil with no possibility of shear failure or excessive settlement. It is the smaller of  $q_{ns}$  and  $q_{np}$ .

### Rankine's Analysis:

Rankine's considered the plastic equilibrium of two adjacent soil elements, one immediately beneath the footing and the other just beyond the edge of the footing. For the element – I beneath the footing, the vertical stress is the major principal stress and the lateral stress is the

minor principal stress. However, for the element – II, the lateral stress becomes the major principal stress and the vertical stress becomes the minor principal stress.



When the loading intensity on the footing approaches the ultimate bearing capacity ( $q_u$ ), the element – I attains a state of plastic equilibrium. However, the element – I can fail only when the adjacent element -II also fails.

For active case

$$\sigma_3' = \sigma_1' \tan^2 \left( 45 - \frac{\Phi'}{2} \right)$$

For element – I

$$\begin{aligned} \sigma_1' &= q_u \\ \therefore \sigma_3' &= q_u \tan^2 \left( 45 - \frac{\Phi'}{2} \right) \end{aligned}$$

For element – II

$$\begin{aligned} \sigma_3' &= \gamma D_f \\ \therefore \sigma_1' &= \frac{\sigma_3'}{\tan^2 \left( 45 - \frac{\Phi'}{2} \right)} = \frac{\gamma D_f}{\tan^2 \left( 45 - \frac{\Phi'}{2} \right)} \end{aligned}$$

As  $\sigma_3'$  of element – I =  $\sigma_1'$  of element – II

$$\begin{aligned} q_u \tan^2 \left( 45 - \frac{\Phi'}{2} \right) &= \frac{\gamma D_f}{\tan^2 \left( 45 - \frac{\Phi'}{2} \right)} \\ q_u &= \gamma D_f \cdot \frac{1}{\tan^4 \left( 45 - \frac{\Phi'}{2} \right)} = \gamma D_f \cdot \tan^4 \left( 45 + \frac{\Phi'}{2} \right) \end{aligned}$$

$$q_u = \gamma D_f \cdot \left( \frac{1 + \sin \Phi'}{1 - \sin \Phi'} \right)^2$$

### Draw backs of Rankine's Analysis:

- Does not give reliable results
- Does not consider cohesion of soil

### Terzaghi's Analysis (Terzaghi's Bearing capacity theory)

#### Assumptions:

1. The base of the footing is rough.
2. The footing is laid at a shallow depth.
3. The shear strength of the soil above the base of the footing is neglected. The soil above the base is replaced by a uniform surcharge  $\gamma D_f$ .
4. The load on the footing is vertical and is uniformly distributed.
5. The footing is long i.e.,  $\frac{L}{B}$  ratio is infinite as the footing is strip footing,
6. The shear strength of the soil is governed by the Mohr - Coulomb equation.

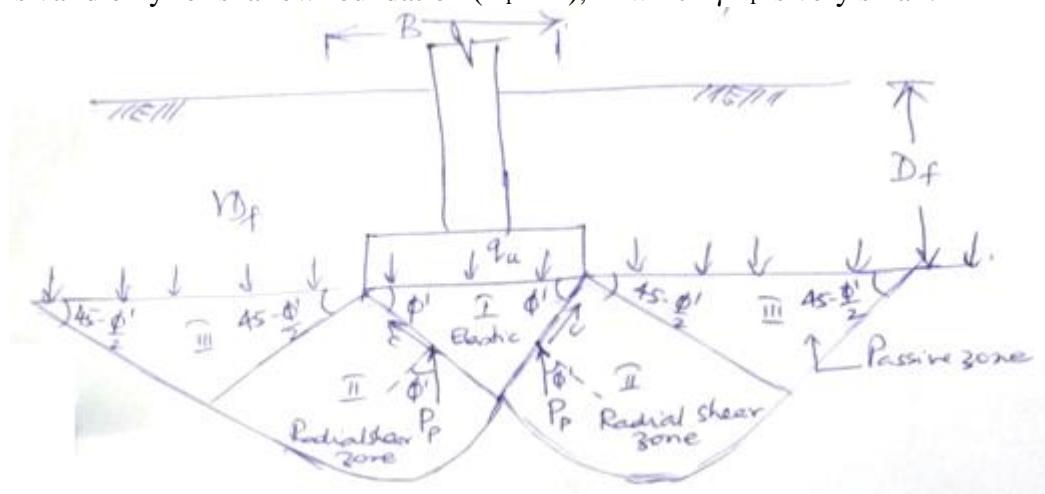
As the base of the footing is rough, the soil in the wedge ABC immediately beneath the footing is prevented from undergoing any lateral yield.

The soil in wedge ABC remains in a state of elastic equilibrium. It behaves as if it were a part of the footing itself. It is assumed that the angles CAB and CBA are equal to the angle of shearing resistance  $\phi'$  of the soil.

The sloping edges AC and BC of the soil wedge CBA bear against the radial shear zones CBD and CAF (zone II). The curves CD and CF are areas of logarithmic spiral.

Two triangular zones BDE and AFG are the Rankine's passive zones (zone III). An overburden pressure  $q = \gamma D_f$  acts as a surcharge on the Rankine's passive zones.

The failure zones do not extend above the horizontal planes passing through the base AB of the footing. i.e., shear resistance of the soil located above the base of the footing is neglected, and the effect of soil is taken equivalent to surcharge  $\gamma D_f$ . Because of this assumption, Terzaghi's theory is valid only for shallow foundation ( $D_f \leq B$ ), in which  $\gamma D_f$  is very small.



The loading conditions are similar to that on a retaining wall under passive pressure case. The failure occurs when the downward pressure exerted by the loads on the soil adjoining the inclined surfaces CB and CA of the soil wedge is equal to the upward pressure.

Downward forces:

- (i)  $q_u x B$
- (ii) Weight of soil wedge =  $\frac{1}{4} \gamma B^2 \tan \Phi'$

Upward forces:

- (i) Vertical components of Resultant passive pressure,  $P_p$
- (ii) Cohesion,  $c'$ , acting along inclined surfaces.

As the resultant passive pressure is inclined at an angle  $\Phi'$  to the normal to the surface of the wedge, it is vertical.

For equilibrium, in vertical direction

$$\frac{1}{4} \gamma B^2 \tan \Phi' + q_u x B = 2 P_p + 2 c' L_i \sin \Phi'$$

Where  $L_i$  = length of the inclined surface CB =  $\frac{B}{2} \cos \Phi'$

$$\therefore q_u x B = 2 P_p + B c' \tan \Phi' - \frac{1}{4} \gamma B^2 \tan \Phi'$$

The resultant passive pressure  $P_p$  has three components

- (i)  $(P_p)_\gamma$  produced by weight of shear zone BCDE, assuming the soil is cohesionless ( $c = 0$ ) and neglecting surcharge  $q$ .
- (ii)  $(P_p)_c$  produced by component  $c'$  of the soil, assuming the soil is weightless and neglecting surcharge.
- (iii)  $(P_p)_q$  produced by surcharge  $q$ , assuming the soil is cohesionless and weightless.

$$\therefore q_u x B = 2 \left[ (P_p)_\gamma + (P_p)_c + (P_p)_q \right] + B c' \tan \Phi' - \frac{1}{4} \gamma B^2 \tan \Phi'$$

Substituting,

$$2(P_p)_\gamma - \frac{1}{4} \gamma B^2 \tan \Phi' = B x \frac{1}{2} \gamma B N_\gamma$$

$$2(P_p)_c + B c' \tan \Phi' = B x c' N_c$$

$$2(P_p)_q = B x \gamma D_f N_q$$

$$q_u x B = B x c' N_c + B x \gamma D_f N_q + B x \frac{1}{2} \gamma B N_\gamma$$

Or

$$q_u = c' N_c + \gamma D_f N_q + 0.5 \gamma B N_\gamma$$

$q_u = c' N_c + q_0 N_q + 0.5 \gamma B N_\gamma$

Ultimate bearing capacity of strip footing under General Shear Failure

Where  $q_0 = \gamma D_f$

$$N_c = \cot \Phi' \left[ \frac{a^2}{2 \cos \left( 45 + \frac{\Phi'}{2} \right)} - 1 \right]$$

$$N_q = \frac{a^2}{2 \cos^2 \left( 45 + \frac{\Phi'}{2} \right)}$$

$$\text{where } a = e^{\left( \frac{3\pi}{4} - \frac{\Phi'}{2} \right) \tan \Phi'}$$

$$N_\gamma = \frac{\tan \Phi'}{2} \left( \frac{K_p}{\cos^2 \Phi'} - 1 \right)$$

$K_p$  = Coefficient of passive earth pressure

For estimating the ultimate bearing capacity of square or circular foundations.

Square footing:

$$q_u = 1.3 c' N_c + q N_q + 0.4 \gamma B N_\gamma$$

Circular footing:

$$q_u = 1.3 c' N_c + q N_q + 0.3 \gamma B N_\gamma$$

For rectangular footing:

$$q_u = c' N_c \left( 1 + 0.2 \frac{B}{L} \right) + q N_q + 0.3 \gamma B N_\gamma \left( 1 - 0.2 \frac{B}{L} \right)$$

Where L = length of the footing.

### Ultimate Bearing Capacity in Case of Local Shear failure

In case of local shear failure

$$C_m' = \frac{2}{3} C'$$

$$\Phi_m' = \tan^{-1} \left( \frac{2}{3} \tan \Phi' \right)$$

The reduced values of  $\Phi' = \Phi_m'$  are used to determine the bearing capacity parameters  $(N_c', N_q' \text{ & } N_\gamma')$  from the value of the General Shear Failure.

$N_q'$  determined from  $\Phi_m'$  given under estimated values. The following expression given by Vesic gives more reliable results:

$$N_q' = \left( e^{3.8 \phi' \tan \phi'} \right) \tan^2 \left( 45 + \frac{\Phi'}{2} \right)$$

The equation for ultimate bearing capacity of strip footing under local shear failure is

$$q_u' = \frac{2}{3} c' N_c' + \gamma D_f N_q' + 0.5 \gamma B N_\gamma'$$

$N_c', N_q' \text{ & } N_\gamma'$  are for reduced values of  $\Phi'$ .

For cohesionless soil, if  $\Phi' > 36^\circ$ , GSF is likely to occur.

If  $\Phi' < 29^\circ$ , LSF is likely to occur.

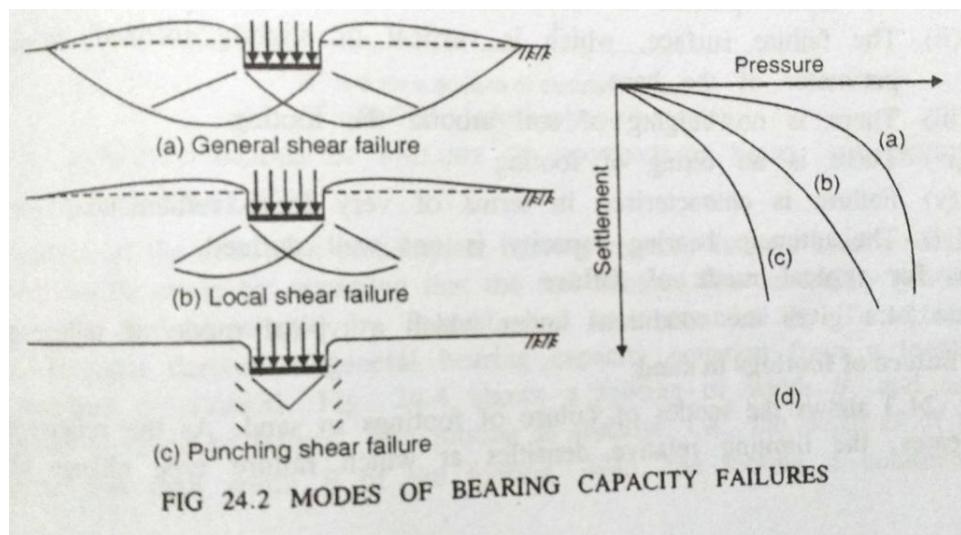
For  $\Phi'$  between  $29^\circ$  and  $36^\circ$ , bearing capacity factors are obtained by interpolation.

	Parameters	GSF	LSF
1	$\Phi$	$> 36^\circ$	$< 29^\circ$
2	Strain at failure	$< 5\%$	No peak in stress-strain curve. Curve rises continuous upto 10-20% strain
3	Relative density, $I_D$	$> 70\%$	$< 35\%$
4	SPT, N	$> 30$	$< 5$
5	Void ratio, e	$< 0.55$	$> 0.75$

## MODES OF SHEAR FAILURE

Depending on the stiffness of foundation soil and depth of foundation, the following are the modes of shear failure experienced by the foundation soil.

1. General shear failure
2. Local shear failure
3. Punching shear failure



### General shear failure

This type of failure is seen in dense and stiff soil. The following are some characteristics of general shear failure.

1. Continuous, well defined and distinct failure surface develops between the edge of footing and ground surface.
2. Dense or stiff soil that undergoes low compressibility experiences this failure.
3. Continuous bulging of shear mass adjacent to footing is visible.
4. Failure is accompanied by tilting of footing.
5. Failure is sudden and catastrophic with pronounced peak in  $P - \Delta$  curve.
6. The length of disturbance beyond the edge of footing is large.
7. State of plastic equilibrium is reached initially at the footing edge and spreads gradually downwards and outwards.

- General shear failure is accompanied by low strain ( $< 5\%$ ) in a soil with considerable  $\Phi$  ( $\Phi > 36^\circ$ ) and large  $N$  ( $N > 30$ ) having high relative density ( $I_D > 70\%$ ).

### Local Shear Failure

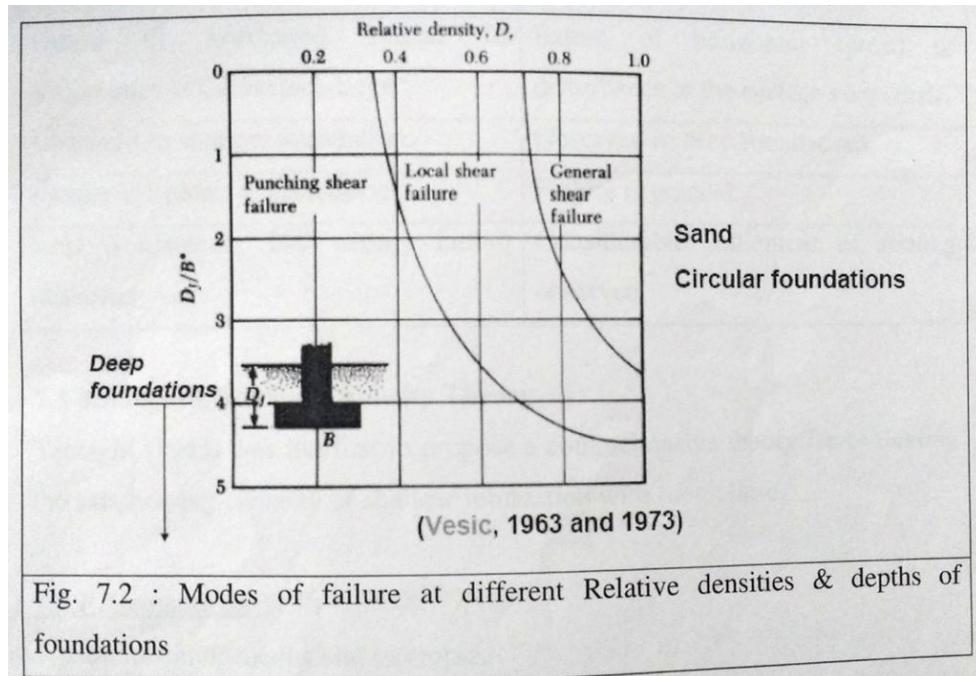
This type of failure is seen in relatively loose and soft soil. The following are some characteristics of local shear failure.

- A significant compression of soil below the footing and partial development of plastic equilibrium is observed.
- Failure is not sudden and there is no tilting of footing.
- Failure surface does not reach the ground surface and slight bulging of soil around the footing is observed.
- Failure surface is not well defined.
- Failure is characterized by considerable settlement.
- Well defined peak is absent in  $P - \Delta$  curve.
- Local shear failure is accompanied by large strain ( $> 10$  to  $20\%$ ) in a soil with considerably low  $\Phi$  ( $\Phi < 28^\circ$ ) and low  $N$  ( $N < 5$ ) having low relative density ( $I_D > 20\%$ ).

### Punching shear failure

This type of failure is seen in loose and soft soil and at deeper elevations. The following are some characteristics of punching shear failure.

- This type of failure occurs in a soil of very high compressibility.
- Failure pattern is not observed.
- Bulging of soil around the footing is absent.
- Failure is characterized by very large settlement.
- Continuous settlement with no increase in  $P$  is observed in  $P - \Delta$  curve.



## Distinction between General Shear and Local or Punching Shear Failures

General Shear Failure	Local / Punching Shear Failure
Occurs in dense / stiff soil $\Phi > 36^\circ$ , $N > 30$ , $I_D > 70\%$ , $C_u > 100 \text{ kPa}$	Occurs in loose / soft soil $\Phi < 28^\circ$ , $N < 5$ , $I_D < 20\%$ , $C_u < 50 \text{ kPa}$
Results in small strain (<5%)	Results in large strain (>20%)
Failure pattern well defined & clear	Failure pattern not well defined
Well defined peak in $P - \Delta$ curve	No peak in $P - \Delta$ curve
Bulging formed in the neighborhood of footing at the surface	No Bulging observed in the neighborhood of footing
Extent of horizontal spread of disturbance at the surface large	Extent of horizontal spread of disturbance at the surface very small
Observed in shallow foundations	Observed in deep foundations
Failure is sudden & Catastrophic	Failure is gradual
Less settlement, but tilting failure observed	Considerable settlement of footing observed

**Ultimate bearing capacity of shallow foundations: square, circular and Rectangular footings – Terzaghi**

### For Rectangular footing

$$(N_c)_r = \left(1 + 0.3 \frac{B}{L}\right) N_c$$

$$(N_\gamma)_r = \left(1 - 0.2 \frac{B}{L}\right) N_\gamma$$

$$(N_q)_r = N_q$$

### **For square footings $\left(\frac{B}{L} = 1\right)$**

$$(N_c)_s = 1.3 N_c$$

$$(N_\gamma)_s = 0.8 N_\gamma$$

$$(N_q)_s = N_q$$

### **For circular footings**

$$(N_c)_c = 1.3 N_c$$

$$(N_\gamma)_c = 0.6 N_\gamma$$

$$(N_q)_c = N_q$$

Using the above shape factors, Terzaghi's equation for strip footing can be approximately used for Rectangular, square or circular footings.

### Depth of shallow foundation (Layered soils- for soft strata)

By Bell's equation

$$D_f = \frac{1}{\gamma} [q K_a^2 - 2c \sqrt{K_a} (1 + K_a)]$$

$q$  = soil pressure at the base of the footing

$K_a$  = active earth pressure coefficient

$c$  = cohesion of the soil

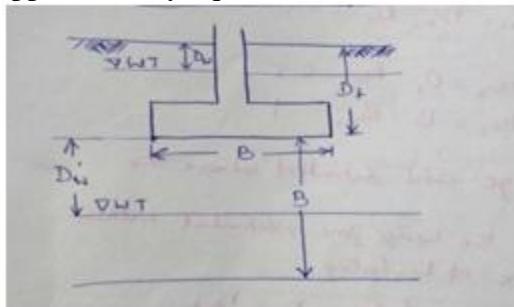
$\gamma$  = unit weight of the soil

### Effect of water table on ultimate bearing capacity

The bearing capacity gets reduced if the WT rises up and comes within a depth  $B$  below the base of the footing. This happens because the soil in the failure zone becomes position of the water table.

$N_q$  term (i.e.,  $\gamma D_f N_q$ ) caused by the surcharge of soil over the footing and  $N_\gamma$  term (i.e.,  $0.5\gamma B N_\gamma$ ) caused by the self weight of the soil **carrying** in the failure wedge are the terms in which the value of  $\gamma$  gets affected.

If the highest level of WT is always below the depth of the shear failure zone, the value of  $\gamma$  in both terms corresponds to the bulk unit weight  $\gamma_t$  of the soil. The depth of the failure zone is approximately equal to the width of the footing  $B$ .



If  $D_w'$  is the depth of water table measured from the base of the footing, for  $D_w' \geq B$ ,  $\gamma = \gamma_t$  in both the terms.

If the water table is at the base of the footing i.e.  $D_w' = 0$ ,  $\gamma'$  should be used in the third term (i.e.  $0.5\gamma B N_\gamma$ )

When  $D_w' \leq B$ , the value of  $\gamma$  to be used is given by effective unit weight,

$$\gamma = \gamma' + \frac{D_w'}{B} (\gamma_t - \gamma')$$

If the water table rises upto the ground surface,  $\gamma'$  is used in both the terms.

If  $D_w$  is the depth of water table measured from the ground surface such that  $0 < D_w < D_f$ ,  $\gamma$  to be used in calculation of  $q (= \gamma D_f)$  is

$$\text{Effective unit weight, } \gamma = \gamma' + \frac{D_w}{D_f} (\gamma_t - \gamma')$$

Also, for this condition,  $\gamma'$  is used in  $N_\gamma$  term ( $0.5\gamma B N_\gamma$ ).

**Alternatively**, Reduction factors can be used.

$$R_q = 0.5 \left( 1 + \frac{D_w}{D_f} \right) \quad (\leq 1) \text{ correction for } N_q$$

$$R_\gamma = 0.5 \left( 1 + \frac{D_w'}{B} \right) \quad (\leq 1) \text{ correction for } N_\gamma$$

### Skempton's Bearing Capacity analysis for clay soils

Skempton's analysis is applicable for saturated clay soil for which  $\Phi_u = 0$ .

Skempton proposed the following expression for  $N_c$ .

$N_c$  increases with the ratio  $\frac{D_f}{B}$ .

#### For strip footings

$$N_c = 5.0 \left( 1 + 0.2 \frac{D_f}{B} \right) \text{ with a max. limiting value of 7.5}$$

#### For square and circular footings

$$N_c = 6.0 \left( 1 + 0.2 \frac{D_f}{B} \right) \text{ with a max. limiting value of 9.0}$$

#### For Rectangular footings

$$N_c = 5.0 \left( 1 + 0.2 \frac{D_f}{B} \right) \left( 1 + 0.2 \frac{B}{L} \right) \text{ for } \frac{D_f}{B} \leq 2.5$$

$$N_c = 7.5 \left( 1 + 0.2 \frac{B}{L} \right) \text{ for } \frac{D_f}{B} > 2.5$$

For  $\Phi_u = 0$ , the net ultimate bearing capacity is given by

$$*q_u = C_u N_c \text{ (approximate } N_c \text{ value to be used)}$$

Unlike Terzaghi's theory, Skepton's analysis can be used for any value of  $\frac{D_f}{B}$ .

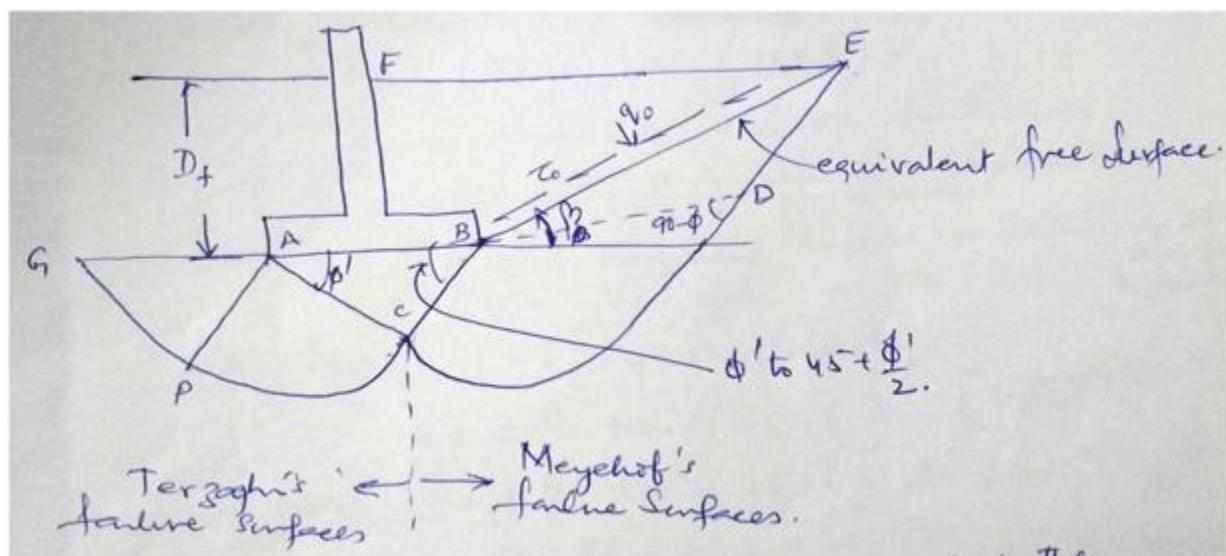
Note: For  $\Phi_u = 0$ ,  $N_q = 1$ ,  $N_y = 0$

### Meyerhof's Bearing Capacity Theory

This theory gives bearing capacity for a strip footing at any depth.

Meyerhof's considered the failure mechanism similar to that assumed by Terzaghi's, but extended the failure surfaces above foundation level.

∴ The shearing strength of the soil above the footing base is also accounted for in the analysis.



Zone ABC is the elastic zone, but the angle which the inclined surfaces AC and BC make with the horizontal varies between  $\phi'$  and  $(45 + \frac{\phi'}{2})$ .

Zone BCD is the zone of Radial Shear.

Zone BDEF is the zone of mixed shear in which shear varies between radial shear and plane shear.

Surface BE is known as "equivalent free surface". It makes an angle  $\beta$  with the horizontal.

The resultant effect of the wedge BEF is represented by normal stress ( $q_0$ ) and shear stress ( $\tau_0$ ) on the surface BE.

$\beta$  increases with  $D_f$  and is equal to  $90^\circ$  for deep foundations.

$\beta$ ,  $q_0$  and  $\tau_0$  are known as foundation depth parameters.

Meyerhof's bearing capacity equation

$$q_u = C N_c s_c d_c i_c + q N_q s_q d_q i_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

s, d and i are empirical corrections factors called shape, depth and inclination factors respectively.

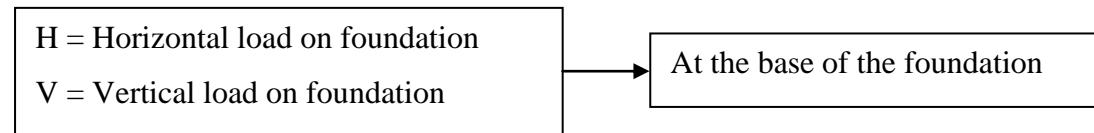
$$N_c = (N_q - 1) \cot \Phi$$

$$N_q = (e^{\pi \tan \Phi}) \tan^2 \left( 45 + \frac{\Phi}{2} \right)$$

$$N_\gamma = (N_q - 1) \tan(1.4\Phi)$$

B = width or dia of foundation.

$$\alpha = \tan^{-1} \left( \frac{H}{V} \right) = \text{inclination of resultant load from the vertical.}$$



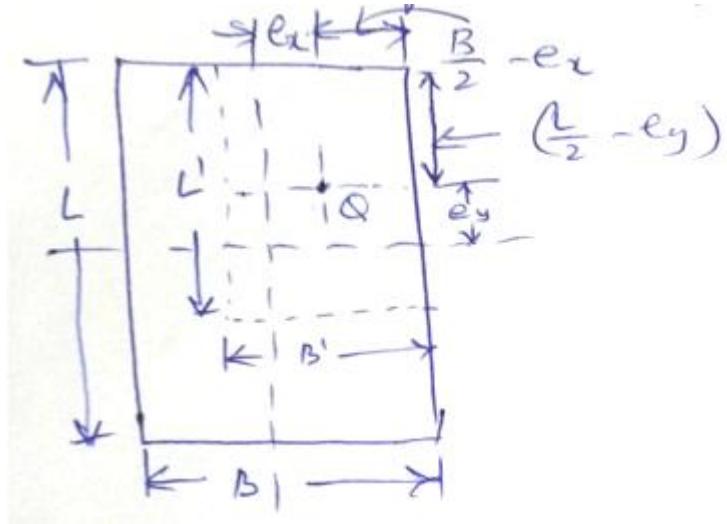
$s_c$ ,  $s_q$  and  $s_\gamma = 1$  for strip footing

$$\begin{aligned}
 s_c &= 1 + 0.2 \frac{B}{L} \tan^2 \left( 45 + \frac{\Phi}{2} \right) \quad \left[ \because K_p = \tan^2 \left( 45 + \frac{\Phi}{2} \right) \right] \\
 s_q, s_\gamma &= 1 + 0.1 \frac{B}{L} \tan^2 \left( 45 + \frac{\Phi}{2} \right) \text{ for } \Phi > 10^\circ \\
 &= 1 \text{ for } \Phi = 0 \\
 d_c &= 1 + 0.2 \frac{D}{B} \tan \left( 45 + \frac{\Phi}{2} \right) \\
 d_q, d_\gamma &= 1 + 0.2 \frac{D}{B} \tan \left( 45 + \frac{\Phi}{2} \right) \text{ for } \Phi > 10^\circ \\
 &= 1 \text{ (for } \Phi = 0) \\
 i_c, i_q &= \left( 1 - \frac{\alpha}{90} \right)^2 \quad \alpha \text{ in degrees} \\
 i_\gamma &= \left( 1 - \frac{\alpha}{\Phi} \right)^2
 \end{aligned}$$

### Useful Width

The concept of useful width is used to compute the bearing capacity when the resultant load on the footing acts eccentrically with respect to the center of the footing.

To account for the eccentricity, the footing dimensions are modified in such a way that the load becomes concentric to the reduced dimensions of the footing.



$$B' = 2 \left( \frac{B}{2} - e_x \right) = B - 2e_x$$

$$L' = 2 \left( \frac{L}{2} - e_y \right) = L - 2e_y$$

$e_x$  = eccentricity in the direction of width

$e_y$  = eccentricity in the direction of length

$$B' = B - 2e_x$$

$$L' = L - 2e_y$$

The effective area  $A'$  for the purpose of calculating the total vertical load  $Q_u$  that can be supported from shear failure consideration is

$$A' = B' \times L'$$

### IS Code (IS 6403-1981) Recommendations for Bearing Capacity

$$q_{nu} = C N_c s_c d_c i_c + q' (N_q - 1) s_q d_q i_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma W'$$

$q'$  = effective surcharge at the base level of the foundation.

$q_{nu}$  = net ultimate Bearing capacity

$$N_c = (N_q - 1) \cot \phi$$

$$N_q = (e^{\pi \tan \phi}) \tan^2 \left( 45 + \frac{\phi}{2} \right)$$

$$N_\gamma = 2(N_q + 1) \tan \phi$$

$W'$  = if water table is likely to remain **permanently** at or below a depth  $(D_f + B)$  below ground level or for  $D_w' \geq B$  where  $D_w'$  is depth of water table measured from the base of the foundation.

In the second term, the influence of water table is taken care of by taking  $q$  as the effective surcharge at the level of the base of the footing.

## Shape, Depth & Inclination factors as per IS: 6403-1981

Factors	Value
$s_c$	$= 1 + 0.2 \frac{B}{L}$ (1.3 for square & circle)
$s_q$	$= 1 + 0.2 \frac{B}{L}$ (1.2 for square & circle)
$s_\gamma$	$= 1 + 0.2 \frac{B}{L}$ for Rectangle , 0.8 for square & 0.6 for circle
$d_c$	$= 1 + 0.2 \frac{D_f}{B} \tan\left(45 + \frac{\Phi}{2}\right)$
$d_q = d_\gamma$	$= 1 + 0.1 \frac{D_f}{B} \tan\left(45 + \frac{\Phi}{2}\right)$ for $\Phi > 10^\circ$ $= 1$ for $\Phi < 10^\circ$

\*Depth factors are used only when backfilling done with proper compaction

$$i_c, i_q = \left(1 - \frac{\alpha}{90}\right)^2 \quad \alpha \text{ in degrees}$$

$$i_\gamma = \left(1 - \frac{\alpha}{\Phi}\right)^2$$

For a cohesionless soil,  $q_{nu}$  of a footing immediately upon construction ( $\Phi_u = 0$ ) is

$$q_{nu} = C_u N_c s_c d_c i_c \quad \text{in which } N_c = 5.14$$

The undrained shear strength  $C_u$  is obtained from unconfined compression strength tests or from correlations with point resistance values.

$$C_u = \frac{q_c}{18} \text{ to } \frac{q_c}{15} \text{ for Normal consolidated clays}$$

$$= \frac{q_c}{26} \text{ to } \frac{q_c}{22} \text{ for Over consolidated clays}$$

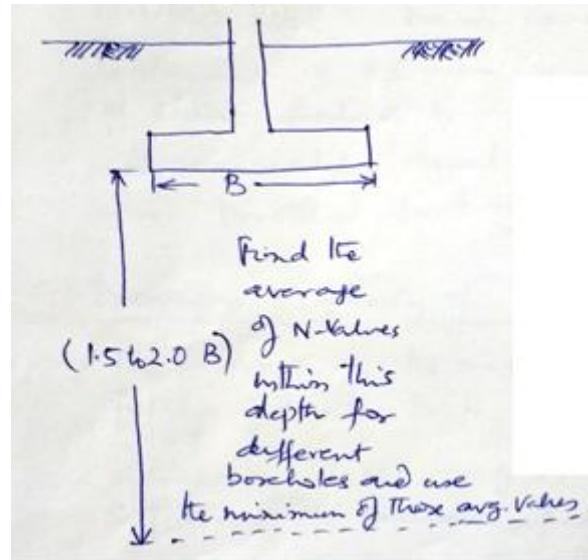
$$q_c = \text{point resistance}$$

## Bearing capacity of Granular soils Based on Standard Penetration Test value

Values of  $N$  are determined at a number of selected locations of bore holes, at vertical intervals of 750 mm or at change of strata, whichever occurs earlier.

The average of  $N$  values (corrected)\* between the level of the base of the footing and a depth equal to 1.5 to 2 times width of the footing below the base, is determined for each of the locations.

\*Note: For dilatancy correction & over burden pressure



### Correlation with N values for cohesionless soils

N – Value	$\Phi^\circ$	Relative Density	Description
< 4	25 – 30	0	Very loose
4 – 10	27 – 32	15	Loose
10 – 30	30 – 35	65	Medium
30 – 50	35 – 40	85	Dense
> 50	38 – 43	100	Very dense

$$\Phi = 25^\circ + 0.15 D_r \text{ (with fines } > 5\%)$$

$$\Phi = 30^\circ + 1.15 D_r \text{ (with fines } < 5\%)$$

N – Value	Unconfined compression strength (kN/m <sup>2</sup> )	Consistency
< 2	25	Very soft
2 – 4	25 – 50	Soft
4 – 8	50 – 100	Medium
8 – 16	100 – 200	Stiff
16 – 32	200 – 400	Very stiff
> 32	> 400	Hard

### Bearing capacity against shear failure in sands from N values (Terzaghi's)

For long (strip or continuous) footings

$$q_{nu} = \frac{1}{6} [5(100 + N^2)D_f R_q + 3 N^2 B R_\gamma] \text{ in kN/m}^2$$

For square or circular footings

$$q_{nu} = \frac{1}{6} [6(100 + N^2)D_f R_q + 2 N^2 B R_\gamma] \text{ in kN/m}^2$$

N = Correlated SPT N value (for overburden and dialatancy)

### Factors influencing Bearing Capacity

$$q_u = qN_q + 0.5 \gamma B N_\gamma \text{ for cohesionless soil (c = 0)}$$

The factors that influence the bearing capacity are

- (i) Relative density or Angle of shearing Resistance
- (ii) Width of the footing
- (iii) Depth of the footing
- (iv) Unit weight of soil
- (v) Position of ground water table

**Relative density:** Higher the Relative Density, greater the angle of shearing resistance  $\Phi'$  of a cohesionless soil.  $N_q$  and  $N_\gamma$  increases with an increase in  $\Phi'$ . Therefore bearing capacity increases.  $N_q$  and  $N_\gamma$  increases appreciably even with a moderate increase in  $\Phi'$  provided the soil deposit possess medium to high relative density. In loose deposit the gain in bearing capacity with the same order of increase of  $\Phi'$  is not significant.

**Width of the footing:**  $q_u$  increases with an increase in the width of the footing, especially when  $\Phi$  is large as  $N_\gamma$  is large. For loose granular soil, increase of width alone will not increase the bearing capacity significantly.

**Depth of the footing:** Increase in depth of foundation will lead to an increase in  $q$  (surcharge) at the base level of the foundation, thus increasing the bearing capacity. Again, increase is significant only when the soil is not a loose deposit.

**Unit weight of soil:** Greater the unit, greater is the bearing capacity.

(Range of unit weight of granular soil: 14 kN/m<sup>3</sup> for a loose, uniform sand to a maximum of about 21 kN/m<sup>3</sup> for mixed grained – sand – boulder mixture).

**Effect of water table:** If water table is at a greater depth, it has no effect on bearing capacity. The bearing capacity could reduce by as much as 50% if the water table rises to the ground level.

### Cohesive soil:

$$q_u = C_u N_c + q$$

Bearing capacity is Not affected by the width of the footing. Increasing depth, increases the surcharge and thereby bearing capacity. Net ultimate bearing capacity is however not affected by the depth of foundation.

When  $\Phi_u = 0$ ,  $N_c = 5.14$  for smooth base and 5.7 for a rough base of the footing.

∴ Bearing capacity of a footing in cohesive soil is dependent mainly on undrained cohesion  $C_u$  of the soil.

## SETTLEMENT OF FOUNDATIONS

Shallow foundations IS: 8009 (Part - I) – 1976;  
Deep foundations IS: 8009 (Part - II) – 1980.

The total settlement of a foundation can be divided into three components.

1. The immediate settlement  $S_i$  which takes place due to elastic deformation of soil without change in water content.
2. The primary consolidation settlement ( $S_c$ )
3. Secondary (creep) settlement which takes place over long periods due to viscous resistance of soil under constant compression.

### Immediate or elastic settlement of footings

Elastic or immediate settlement of a flexible footing placed on the surface is given by

$$S_i = \frac{qB(1-\mu^2)}{E} I_f \quad \text{For foundation at surface}$$

$q$	= net foundation pressure
$B$	= width of the foundation
$E$	= Young's Modulus
$\mu$	= Poisson's ratio
$I_f$	= Influence factor

Example of flexible foundation: Isolated footing

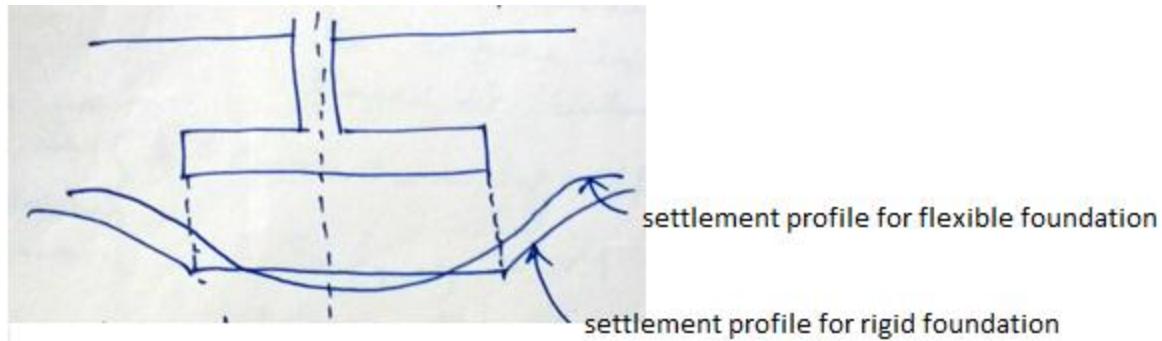
Example of Rigid foundation: Raft or Mat foundation

$I_f$  depends on shape of foundation and rigidity of foundation

### Use of elastic theory

1. The soil is assumed to be semi infinite or layered elastic solid.
2.  $E$  &  $\mu$  are assumed constant

### Settlement Response of foundation to applied loading



**Note:**

1. Settlement at corner is not equal to that at center for flexible foundation.
2. Settlement under the footing is uniform throughout. Contact pressure more near edges and less near the center, in order to produce uniform settlement.

In a flexible footing, contact pressure is uniform. Uniform pressure produces a dish shaped pattern of displacement.

**Note:**

1. For flexible foundation  $I_f$  is taken as average of that at the center and the corner.
2.  $I_f$  (Rigid) = 0.8  $I_f$  (flexible)

**Rigidity correction**

Influence factor for Rigid foundation = 0.8 x Influence factor of flexible foundation at center.

Even for a rigid foundation, First calculate the  $I_f$  of the flexible foundation at the center and then apply the rigidity correction as shown above.

**Consolidation settlements ( $S_c$ )**

$$S_c = \frac{C_c}{1 + e_0} H \log_{10} \left( \frac{p_0 + \Delta p}{p_0} \right) \quad \text{or} \quad S_c = m_v H \Delta p$$

$\bar{p}_0$	= initial effective overburden pressure before applying foundation load.
$\Delta p$	= vertical stress at the center due to applied load
$e_0$	= Initial void ratio
$H$	= Thickness of soil layer
$C_c$	= compression index

### Corrections to be applied (to consolidation settlement)

1	<p><b>Rigidity correction for a rigid foundation</b></p> $S_{(rigid)} = 0.8 \times S_{center \ (flexible)}$ <p style="text-align: center;">↑</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>According to IS: 8009 (Part – 1) – 1976, correction factor or Rigidity factor.</p> </div> <div style="text-align: center; margin-top: 20px;"> <math display="block">\text{Rigidity factor} = \frac{\text{total settlement of Rigid foundation}}{\text{total settlement of flexible foundation}}</math> </div>
2	<p><b>Depth correction</b></p> <p>Correction factor or Depth factor = <math>\frac{S_{(embedded)}}{S_{(surface)}}</math> i.e., the settlement for the footing placed at the surface is corrected for the footing placed below the ground surface.</p>
*Note:	1 & 2 are applies to both immediate and consolidation settlement
3	<p><b>Correction for effect of 3-D consolidation only</b></p> $S_{c(3-D)} = \mu S_{c(1-D)}$ <p>Where <math>\mu</math> = correction factor (obtained from standard graph)</p>

One of the basic criteria governing design of foundations is that the settlement must not exceed the permissible value.

(a) Immediate settlement or elastic settlement ( $S_i$ ) takes place immediately or in a short time (less than about 7 days) after a load is placed.

In clay soil, this is also known as distortion settlement and is due to change in the shape of soil without a change in volume or water content.

In saturated clay, immediate settlement is computed using the elastic theory. It is sometimes considered small compared to the long – term consolidation settlement and, therefore neglected.

(b) **Primary Consolidation Settlement ( $S_c$ )** which is due to gradual expulsion of pore water from the voids of the soil, resulting in dissipation of excess pore water pressure and an increase in effective stress.

This is computed using Terzaghi's theory of one-dimensional consolidation. This is appropriate for all nearly saturated fine grained soils.

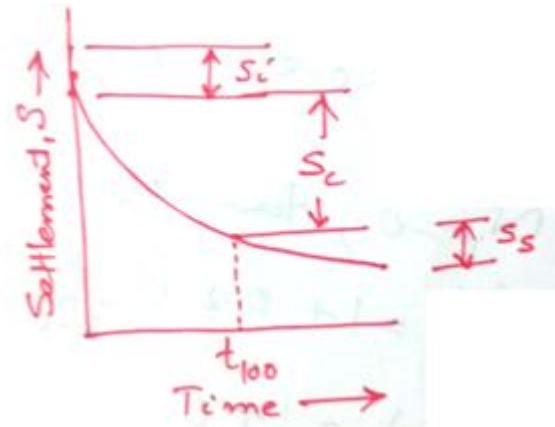
(c) **Secondary compression settlement ( $S_s$ )** which occurs at constant effective stress, with volume change occurring due to rearrangement of soil particles.

This is also known as creep settlement. This settlement becomes important for certain types of soils, such as peats and soft organic clays.

For stiff clays or preconsolidated clays, this component is relatively minor.

The total vertical settlement,  $S_t$ , in a general case is thus,

$$S_t = S_i + S_c + S_s$$



Difficulty in using elastic theory is correct evaluation of  $E$  and  $\mu$ . In a saturated clay of wide extent, subjected to rapid loading locally deformation takes place under constant volume condition, for which  $\mu = 0.5$

$E$  is determined as secant modulus obtained from undrained triaxial test over a range of stress from zero to  $\frac{1}{2}q_u$  (ultimate bearing capacity).

### Granular soils

In granular soils,  $E$  increases with confining pressure, and therefore increases with depth.

$E$  also varies across width of loaded area, being greater near the center than near the edges.

The settlement in a cohesionless soil is due to both elastic and primary compression. Settlements in such soils are determined using either semi empirical approaches or field tests.