



N.B.K.R. INSTITUTE OF SCIENCE & TECHNOLOGY

(AUTONOMOUS)

Affiliated to JNTU, Anantapuram

eXPERTS

FLUID MeCHANICS-I

e-LEARNING

For

II B.Tech CIVIL ENGINEERING

2016-2017 Admitted Batch

UNIT – I

FLUID PROPERTIES : Definition of a fluid – Density, Specific weight, Specific volume, Specific gravity – Viscosity – Bulk modulus of elasticity – Vapour pressure – Surface tension and capillarity – Continuum.

UNIT – II

FLUID STATICS : Pressure at a point – Absolute and guage pressures – Pascal's and Hydrostatic laws – Pressure measurement – Manometers and mechanical gauges – Hydrostatic thrust on plane and curved surfaces – Buoyancy and flotation –Metacentric height.

UNIT – III

FLUID FLOW CONCEPTS: Flow characteristics – Velocity – acceleration – Types of flow – Streamlines, path lines, streak lines – stream function, velocity potential, flownet – circulation and Vorticity.

UNIT – IV

FUNDAMENTAL EQUATIONS: Continuity equation – Euler's equation of motion along a streamline – Bernoulli's equation –Linear momentum equation – Forces on a bend – Fixed and moving vanes – Moment of momentum equation – Torque on sprinklers.



UNIT – V

FLOW MEASUREMENT: Velocity measurement – Pitot tube – Pitot Static tube – Discharge measurement – Orifices and Mouth pieces – Venturimeter, Nozzlemeter, Orificemeter, Notches and Weirs.

DIMENSIONAL ANALYSIS AND SIMILITUDE: Dimensional homogeneity – Methods of dimensional analysis – Model investigations – Similitude – Dimensionless numbers – Model laws – Undistorted and distorted models – Scale effects.



UNIT - I

FLUID PROPERTIES



CONTENTS

- Definition of a fluid
- Density, Specific weight, Specific volume, Specific gravity
- Viscosity
- Bulk modulus of elasticity
- Vapour pressure
- Surface tension and capillarity
- Continuum.

INTRODUCTION

Fluid mechanics in everyday life

There is air around us, and there are rivers and seas near us. 'The flow of a river never ceases to go past, nevertheless it is not the same water as before. Bubbles floating along on the stagnant water now vanish and then develop but have never remained.' So stated ChohmeiKamo, the famous thirteenth-century essayist of Japan, in the prologue of Hohjohki, his collection of essays. In this way, the air and the water of rivers and seas are always moving. Such a movement of gas or liquid (collectively called 'fluid') is called the 'flow', and the study of this is 'fluid mechanics'. While the flow of the air and the water of rivers and seas are flows of our concern, so also are the flows of water, sewage and gas in pipes, in irrigation canals, and around rockets, aircraft, express trains, automobiles and boats. And so too is the resistance which acts on such flows. Throwing baseballs and hitting golf balls are all acts of flow. Furthermore, the movement of people on the platform of a railway station or at the intersection of streets can be regarded as forms of flow. In a wider sense, the movement of social phenomena, information or history could be regarded as a flow, too. In this way, we are in so close a relationship to flow that the 'fluid mechanics' which studies flow is really a very familiar thing to us.



A Brief Look Back in History

Before proceeding with our study of fluid mechanics, we should pause for a moment to consider the history of this important engineering science. As is true of all basic scientific and engineering disciplines, their actual beginnings are only faintly visible through the haze of early antiquity. But, we know that interest in fluid behavior dates back to the ancient civilizations. Through necessity there was a practical concern about the manner in which spears and arrows could be propelled through the air, in the development of water supply and irrigation systems, and in the design of boats and ships. These developments were of course based on trial and error procedures without any knowledge of mathematics or mechanics. However, it was the accumulation of such empirical knowledge that formed the basis for further development during the emergence of the ancient Greek civilization and the subsequent rise of the Roman Empire. Some of



the earliest writings that pertain to modern fluid mechanics are those of Archimedes 1287–212 B.C.², a Greek mathematician and inventor who first expressed the principles of hydrostatics and flotation. Elaborate water supply systems were built by the Romans during the period from the fourth century B.C. through the early Christian period, and Sextus Julius Frontinus 1A.D. 40–1032, a Roman engineer, described these systems in detail. However, for the next 1000 years during the Middle Ages (also referred to as the Dark Ages), there appears to have been little added to further understanding of fluid behavior.

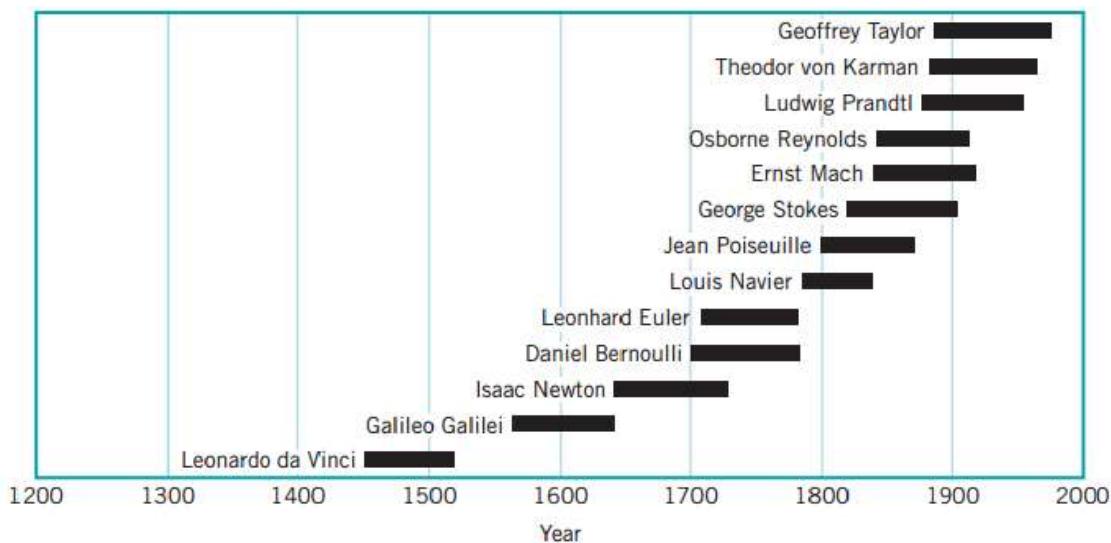


FIGURE 1.1 Time line of some contributors to the science of fluid mechanics

As shown in Fig. 1.1, beginning with the Renaissance (period about the fifteenth century) a rather continuous series of contributions began that forms the basis of what we consider to be the science of fluid mechanics. Leonardo da Vinci 1452–15192 described through sketches and writings many different types of flow phenomena. The work of Galileo Galilei 1564–16422 marked the beginning of experimental mechanics. Following the early Renaissance period and during the seventeenth and eighteenth centuries, numerous significant contributions were made. These include theoretical and mathematical advances associated with the famous names of Newton, Bernoulli, Euler, and d'Alembert. Experimental aspects of fluid mechanics were also advanced during this period, but unfortunately the two different approaches, theoretical and experimental, developed along separate paths. *Hydrodynamics* was the term associated with the



theoretical or mathematical study of idealized, frictionless fluid behavior, with the term *hydraulics* being used to describe the applied or experimental aspects of real fluid behavior, particularly the behavior of water. Further contributions and refinements were made to both theoretical hydrodynamics and experimental hydraulics during the nineteenth century, with the general differential equations describing fluid motions that are used in modern fluid mechanics being developed in this period. Experimental hydraulics became more of a science, and many of the results of experiments performed during the nineteenth century are still used today.

At the beginning of the twentieth century, both the fields of theoretical hydrodynamics and experimental hydraulics were highly developed, and attempts were being made to unify the two. In 1904 a classic paper was presented by a German professor, Ludwig Prandtl 1875–1953, who introduced the concept of a “fluid boundary layer,” which laid the foundation for the unification of the theoretical and experimental aspects of fluid mechanics. Prandtl’s idea was that for flow next to a solid boundary a thin fluid layer (boundary layer) develops in which friction is very important, but outside this layer the fluid behaves very much like a frictionless fluid. This relatively simple concept provided the necessary impetus for the resolution of the conflict between the hydrodynamicists and the hydraulicists. Prandtl is generally accepted as the founder of modern fluid mechanics. Also, during the first decade of the twentieth century, powered flight was first successfully demonstrated with the subsequent vastly increased interest in *aerodynamics*. Because the design of aircraft required a degree of understanding of fluid flow and an ability to make accurate predictions of the effect of air flow on bodies, the field of aerodynamics provided a great stimulus for the many rapid developments in fluid mechanics that took place during the twentieth century.



As we proceed with our study of the fundamentals of fluid mechanics, we will continue to note the contributions of many of the pioneers in the field. Table 1 provides a chronological listing of some of these contributors and reveals the long journey that makes up the history of fluid mechanics. This list is certainly not comprehensive with regard to all of the past contributors, but includes those who are mentioned in this text.



As mention is made in succeeding chapters of the various individuals listed in Table 1, a quick glance at this table will reveal where they fit into the historical chain. It is, of course, impossible to summarize the rich history of fluid mechanics in a few paragraphs. Only brief glimpses are provided, and hope it will stir your interest.

TABLE 1 Chronological Listing of Some Contributors to the Science of Fluid Mechanics

ARCHIMEDES 1287–212 B.C.2	LEONHARD EULER 1707–17832
Established elementary principles of buoyancy and flotation.	First explained role of pressure in fluid flow; formulated basic equations of motion and so-called Bernoulli theorem; introduced concept of cavitation and principle of centrifugal machinery.
SEXTUS JULIUS FRONTINUS 1 A.D. 40–1032	JEAN le ROND d'ALEMBERT 1717–17832
Wrote treatise on Roman methods of water distribution.	Originated notion of velocity and acceleration components, differential expression of continuity, and paradox of zero resistance to steady non-uniform motion.
LEONARDO da VINCI 1452–15192	ANTOINE CHEZY 1718–17982
Expressed elementary principle of continuity; observed and sketched many basic flow phenomena; suggested designs for hydraulic machinery.	Formulated similarity parameter for predicting flow characteristics of one channel from measurements on another.
GALILEO GALILEI 1564–16422	GIOVANNI BATTISTA VENTURI 1746–18222
Indirectly stimulated experimental hydraulics; Revised Aristotelian concept of vacuum.	Performed tests on various forms of mouthpieces — in particular, conical contractions and expansions.
EVANGELISTA TORRICELLI 1608–16472	LOUIS MARIE HENRI NAVIER 1785–18362
Related barometric height to weight of atmosphere, and form of liquid jet to trajectory of free fall.	Extended equations of motion to include “molecular” forces.
BLAISE PASCAL 1623–16622	AUGUSTIN LOUIS de CAUCHY 1789–18572
Finally clarified principles of barometer, hydraulic press, and pressure transmissibility.	Contributed to the general field of theoretical hydrodynamics and to the study of wave motion.
ISAAC NEWTON 1642–17272	GOTTHILF HEINRICH LUDWIG HAGEN
Explored various aspects of fluid resistance — inertial, viscous, and wave; discovered jet contraction.	1797–18842 Conducted original studies of resistance in and transition between laminar and turbulent flow.
HENRI de PITOT 1695–17712	JEAN LOUIS POISEUILLE 1799–18692
Constructed double-tube device to indicate water velocity through differential head.	Performed meticulous tests on resistance of flow through capillary tubes.
DANIEL BERNOULLI 1700–17822	HENRI PHILIBERT GASPARD DARCY 17803–18582
Experimented and wrote on many phases of fluid motion, coining name “hydrodynamics”; devised manometry technique and adapted primitive energy principle to explain velocity-head indication; proposed jet propulsion.	Performed extensive tests on filtration and pipe



8 | UNIT – I : FLUID PROPERTIES

resistance; initiated open-channel studies carried out by Bazin.

JULIUS WEISBACH 11806–18712

Incorporated hydraulics in treatise on engineering mechanics, based on original experiments; noteworthy for flow patterns, nondimensional coefficients, weir, and resistance equations.

WILLIAM FROUDE 11810–18792

Developed many towing-tank techniques, in particular the conversion of wave and boundary layer resistance from model to prototype scale.

ROBERT MANNING 11816–18972

Proposed several formulas for open-channel resistance.

GEORGE GABRIEL STOKES 11819–19032

Derived analytically various flow relationships ranging from wave mechanics to viscous resistance—particularly that for the settling of spheres.

ERNST MACH 11838–19162

One of the pioneers, in the field of supersonic aerodynamics.

OSBORNE REYNOLDS 11842–19122

Described original experiments in many fields—cavitation, river model similarity, pipe resistance—and devised two parameters for viscous flow; adapted equations of motion of a viscous fluid to mean conditions of turbulent flow.

JOHN WILLIAM STRUTT, LORD RAYLEIGH

11842–19192

Investigated hydrodynamics of bubble collapse,

wave motion, jet instability, laminar flow analogies, and dynamic similarity.

VINCENZ STROUHAL 11850–19222

Investigated the phenomenon of “singing wires.”

EDGAR BUCKINGHAM 11867–19402

Stimulated interest in the United States in the use of dimensional analysis.

MORITZ WEBER 11871–19512

Emphasized the use of the principles of similitude in fluid flow studies and formulated a capillarity similarity parameter.

LUDWIG PRANDTL 11875–19532

Introduced concept of the boundary layer and is generally considered to be the father of present-day fluid mechanics.

LEWIS FERRY MOODY 11880–19532

Provided many innovations in the field of hydraulic machinery. Proposed a method of correlating pipe resistance data which is widely used.



THEODOR VON KÁRMÁN 11881–19632

One of the recognized leaders of twentieth century fluid mechanics. Provided major contributions to our understanding of surface resistance, turbulence, and wake phenomena.

PAUL RICHARD HEINRICH BLASIUS

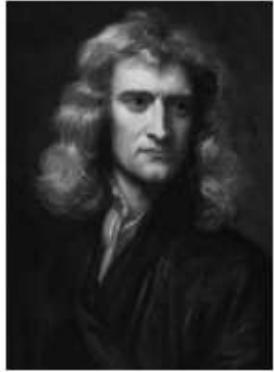
11883–19702

One of Prandtl's students who provided an analytical solution to the boundary layer equations. Also, demonstrated that pipe resistance was related to the Reynolds number.





Leonardo da Vinci



Isaac Newton



Daniel Bernoulli



Ernst Mach



Osborne Reynolds



Ludwig Prandtl



UNIT – I

FLUID PROPERTIES

Lecture Hours:

Content: Definition of a fluid – Density, Specific weight, Specific volume, Specific gravity – Viscosity – Bulk modulus of elasticity – Vapour pressure – Surface tension and capillarity – Continuum.

OBJECTIVES

When you finish reading this chapter, you should be able to

- Have a working knowledge of the basic properties of fluids and understand the continuum approximation.
- Have a working knowledge of viscosity and the consequences of the frictional effects it causes in fluid flow.
- Calculate the capillary rises and drops due to the surface tension effect.

What Is a Mechanics?



Mechanics is the oldest physical science that deals with both stationary and moving bodies under the influence of forces. The branch of mechanics that deals with bodies at rest is called **statics**, while the branch that deals with bodies in motion is called **dynamics**. The sub-category **fluid mechanics** is defined as the science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics), and the interaction of fluids with solids or other fluids at the boundaries. Fluid mechanics is also referred to as **fluid dynamics** by considering fluids at rest as a special case of motion with zero velocity.

Fluid mechanics itself is also divided into several categories. The study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **hydrodynamics**. A sub-category of hydrodynamics is **hydraulics**, which deals with liquid flows in pipes and open channels. **Gas dynamics** deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds. The category **aerodynamics** deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds. Some other specialized categories such as **meteorology**, **oceanography**, and **hydrology** deal with naturally occurring flows.



What Is a Fluid?

You will recall from physics that a substance exists in three primary phases: solid, liquid, and gas. (At very high temperatures, it also exists as plasma.) A substance in the liquid or gas phase is referred to as a **fluid**. Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape. A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small. In solids stress is proportional to strain, but in fluids stress is proportional to strain rate. When a constant shear force is applied, a solid eventually stops deforming, at some fixed strain angle, whereas a fluid never stops deforming and approaches a certain rate of strain.

You will recall from statics that **stress** is defined as force per unit area and is determined by dividing the force by the area upon which it acts. The normal component of the force acting on a surface per unit area is called the **normal stress**, and the tangential component of the force acting on a surface per unit area is called **shear stress**. In a fluid at rest, the normal stress is called **pressure**.

Application Areas of Fluid Mechanics

Fluid mechanics is widely used both in everyday activities and in the design of modern engineering systems from vacuum cleaners to supersonic aircraft. Therefore, it is important to develop a good understanding of the basic principles of fluid mechanics. To begin with, fluid mechanics plays a vital role in the human body. The heart is constantly pumping blood to all parts of the human body through the arteries and veins, and the lungs are the sites of airflow in alternating directions. Needless to say, all artificial hearts, breathing machines, and dialysis systems are designed using fluid dynamics. An ordinary house is, in some respects, an exhibition hall filled with applications of fluid mechanics. The piping systems for cold water, natural gas, and sewage for an individual house and the entire city are designed primarily on the basis of fluid mechanics. The same is also true for the piping and ducting network of heating and air-conditioning systems. A refrigerator involves tubes, through which the refrigerant flows, a compressor that pressurizes the refrigerant, and two heat exchangers where the refrigerant absorbs and rejects heat. Fluid mechanics plays a major role in the design of all these components. Even the operation of ordinary faucets is based on fluid mechanics. We can also see numerous applications of fluid mechanics in an automobile. All components associated with the transportation of the fuel from the fuel tank to the cylinders—the fuel line, fuel pump, fuel injectors, or carburetors—as well as the mixing of the fuel and the air in the cylinders and the purging of combustion gases in exhaust pipes are analyzed using fluid mechanics. Fluid mechanics is also used in the design of the heating and air-conditioning system, the hydraulic brakes, the power steering, automatic transmission, lubrication systems and the cooling system of the engine.



block including the radiator and the water pump, and even the tires. The sleek streamlined shape of recent model cars is the result of efforts to minimize drag by using extensive analysis of flow over surfaces. On a broader scale, fluid mechanics plays a major part in the design and analysis of aircraft, boats, submarines, rockets, jet engines, wind turbines, biomedical devices, the cooling of electronic components, and the transportation of water, crude oil, and natural gas. It is also considered in the design of buildings, bridges, and even billboards to make sure that the structures can withstand wind loading. Numerous natural phenomena such as the rain cycle, weather patterns, rise of ground water to the top of trees, winds, ocean waves, and currents in large water bodies are also governed by the principles of fluid mechanics.



Natural flows and weather
© Vol. 16/Photo Disc.



Boats
© Vol. 5/Photo Disc.



Aircraft and spacecraft
© Vol. 1/Photo Disc.



Power plants
© Vol. 57/Photo Disc.



Human body
© Vol. 110/Photo Disc.



Cars
Photo by John M. Cimbala.



Wind turbines
© Vol. 17/Photo Disc.



Piping and plumbing systems
Photo by John M. Cimbala.



Industrial applications
Courtesy UMDE Engineering, Contracting, and Trading. Used by permission.

FIGURE 1.1 Some application areas of fluid mechanics.

Three Phases of Matter

Generally matter exists in three phases namely (i) Solid (ii) Liquid and (iii) Gas (includes vapour). The last two together are also called by the common term **fluids**. In solids atoms/molecules are closely spaced and the attractive (cohesive) forces between atoms/molecules are high. The shape is maintained by the cohesive forces binding the



atoms. When an external force is applied on a solid component, slight rearrangement in atomic positions balances the force. Depending upon the nature of force the solid may elongate or shorten or bend. When the applied force is removed the atoms move back to the original position and the former shape is regained. Only when the forces exceed a certain value (yield), a small deformation called plastic deformation will be retained as the atoms are unable to move to their original positions. When the force exceeds a still higher value (ultimate), the cohesive forces are not adequate to resist the applied force and the component will break.

In liquids, inter molecular distances are longer and the cohesive forces are of smaller in magnitude. The molecules are not bound rigidly as in solids and can move randomly. However, the cohesive forces are large enough to hold the molecules together below a free surface that forms in the container. Liquids will continue to deform when a shear or tangential force is applied. The deformation continues as long as the force exists. In fluids the **rate of deformation** controls the force (not deformation as in solids). More popularly it is stated that a fluid (liquid) cannot withstand applied shear force and will continue to deform. When at rest, liquids will assume the shape of the container forming a free surface at the top.

In gases the distance between molecules is much larger compared to atomic dimensions and the cohesive force between atoms/molecules is low. So gas molecules move freely and fill the full volume of the container. If the container is open the molecules will diffuse to the outside. Gases also cannot withstand shear. The **rate of deformation** is proportional to the applied force as in the case of liquids.

Liquids and gases together are classified as fluids. Vapour is gaseous state near the evaporation temperature. The state in which a material exists depends on the pressure and temperature. For example, steel at atmospheric temperature exists in the solid state. At higher temperatures it can be liquefied. At still higher temperatures it will exist as a vapour.

A fourth state of matter is its existence as charged particles or ions known as plasma.

Compressible and Incompressible Fluids

If the density of a fluid varies significantly due to moderate changes in pressure or temperature, then the fluid is called **compressible fluid**. Generally gases and vapours under normal conditions can be classified as compressible fluids. In these phases the distance between atoms or molecules is large and cohesive forces are small. So, increase in pressure or temperature will change the density by a significant value.



If the change in density of a fluid is small due to changes in temperature and/or pressure, then the fluid is called **incompressible fluid**. All liquids are classified under this category.

When the change in pressure and temperature is small, gases and vapours are treated as incompressible fluids. For certain applications like propagation of pressure disturbances, liquids should be considered as compressible.

In this chapter some of the properties relevant to fluid mechanics are discussed with a view to bring out their influence on the design and operation of fluid machinery and equipment.

Continuum

Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as a continuous, homogeneous matter with no holes, that is, a **continuum**. The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities. This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules. This is the case in practically all problems, except some specialized ones. The continuum idealization is implicit in many statements we make, such as “the density of water in a glass is the same at any point.”



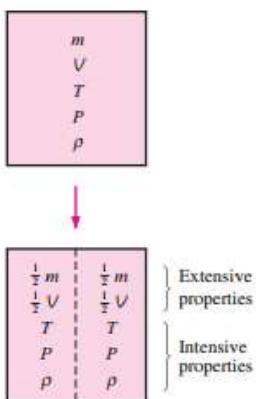


FIG 1.2 System of imaginary partition

Properties of Fluids

Any characteristic of a system is called a **property**. Some familiar properties are pressure P , temperature T , volume V , and mass m . The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation. Properties are considered to be either intensive or extensive. **Intensive properties** are those that are independent of the mass of a system, such as temperature, pressure, and density. **Extensive properties** are those whose values depend on the size - or extent - of the system.

FIG. 1.2. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.

Generally, uppercase letters are used to denote extensive properties (with mass m being a major exception), and lowercase letters are used for intensive properties (with pressure P and temperature T being the obvious exceptions). Extensive properties per unit mass are called **specific properties**. Some examples of specific properties are specific volume ($v = V/m$) and specific total energy ($e = E/m$).



The state of a system is described by its properties. But we know from experience that we do not need to specify all the properties in order to fix a state. Once the values of a sufficient number of properties are specified, the rest of the properties assume certain values. That is, specifying a certain number of properties is sufficient to fix a state. The number of properties required to fix the state of a system is given by the **state postulate**: *The state of a simple compressible system is completely specified by two independent, intensive properties*.

Density or Mass Density.

- Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (rho).
- The unit of mass density in SI unit is kg per cubic metre. i.e., kg/m^3 . The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.
- Mathematically, mass density is written as



$$\rho \text{ (rho)} = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

- The value of density of water is 1 gm/cm^3 or 1000 kg/m^3 .

Specific Weight or Weight Density

- Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol **w** or **Y (gamma)**.
- Thus mathematically,

$$\begin{aligned} w &= \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}} \\ &= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}} = \rho * g \\ \therefore w &= \rho * g \quad \text{----- (1)} \end{aligned}$$

The value of specific weight or weight density (w) for water is $9.81 \times 1000 \text{ Newton/m}^3$ in SI units.

Specific Volume.

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume of fluid}}} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density.

It is expressed as m^3/kg . It is commonly applied to gases.



Specific Gravity

Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken **water** and for gases, the standard fluid is taken **air**.

Specific gravity is also called **relative density**.

It is dimensionless quantity and is denoted by the symbol **S**.

Mathematically,

$$S \text{ (for liquid or gas)} = \frac{\text{Weight density of liquid or gas}}{\text{Weight density of water or air}}$$

Thus the weight density of a liquid = $S * \text{Weight density of water}$



$$= S * 1000 * 9.81 \text{ N/m}^3$$

The density of a liquid

$$= S * \text{Density of water} = S * 1000 \text{ kg/m}^3$$

If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water.

For example, the specific gravity of mercury is 13.6, hence density of mercury = $13.6 \times 1000 = 13600 \text{ kg/m}^3$.

Viscosity

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance ' dy ' apart, move one over the other at different velocities, say u and $u + du$ as shown in **FIG. 1.3**, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y . It is denoted by symbol **τ (Tau)**.

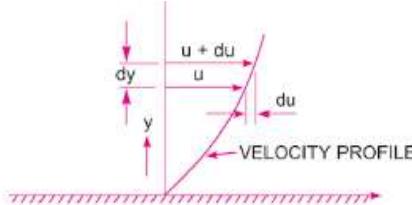


FIG. 1.3 Velocity variation near a soil boundary

Mathematically,

$$\boxed{\tau \propto \frac{du}{dy} \text{ or } \mu^* \frac{du}{dy}} \quad \text{--- (2)}$$

Where μ (called **μ**) is the constant of proportionality and is known as the coefficient of *dynamic viscosity or only viscosity* $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

$$\text{From equation (2), we have } \mu = \frac{\tau}{\frac{du}{dy}} \quad \text{--- (3)}$$

Thus viscosity is also defined as the shear stress required to produce, unit rate of shear strain.

Units of Viscosity.



The units of viscosity is obtained by putting the dimensions of the quantities in equation (3)

$$\mu = \frac{\text{Shear Stress}(\tau)}{\frac{\text{Change of velocity } (du)}{\text{Change of distance } (dy)}} = \frac{\frac{\text{Force}}{\text{Area}}}{\left(\frac{\text{Length}}{\text{Time}}\right) * \frac{1}{\text{Length}}} = \frac{\frac{\text{Force}}{\text{Length}^2}}{\frac{1}{\text{Time}}} = \frac{\text{Force} * \text{Time}}{\text{Length}^2}$$

$$\therefore \text{MKS units of viscosity} = \frac{\text{kgs-sec}}{\text{m}^2}$$

$$\text{CGS units of viscosity} = \frac{\text{dyne-sec}}{\text{cm}^2}$$

$$\text{SI units of viscosity} = \frac{\text{N-sec}}{\text{m}^2} = \text{Pa.s}$$

The unit of viscosity in CGS is also called Poise which is equal to $\frac{\text{dyne-sec}}{\text{cm}^2}$

$$\therefore \frac{\text{one kgs-sec}}{\text{m}^2} = \frac{9.81 \text{Ns}}{\text{m}^2} = \text{98.1 poise}$$

$$\therefore \text{One poise} = \frac{1}{10} \frac{\text{Ns}}{\text{m}^2}$$

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise} \text{ or } 1 \text{ cP} = \frac{1}{100} \text{ P.}$$

Kinematic Viscosity



It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol ν (called **nu**). Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\frac{\text{Force} * \text{Time}}{\text{Length}^2} \frac{\text{Length}^2}{\text{Time}}}{\frac{\text{Length}^2 * \frac{\text{Mass}}{\text{Length}^3}}{\text{Time}}} = \frac{\text{Force} * \text{Time}}{\text{Mass} * \text{Length}}$$

$$\text{Thus, One Stoke} = \text{cm}^2/\text{sec} = \left(\frac{1}{100}\right)^2 \text{ m}^2/\text{sec} = 10^{-4} \text{ m}^2/\text{s}$$

Centistoke = 1/100 stoke.

Newton's Law of Viscosity

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity. Mathematically, it is expressed as given by **equation (2)**.



Types of Fluids

The fluids may be classified into the following five types:

1. Ideal fluid,
2. Real fluid.
3. Newtonian fluid,
4. Non-Newtonian fluid and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and having no viscosity, is known as an Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice are real fluids.

3. Newtonian Fluid. A real fluid in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

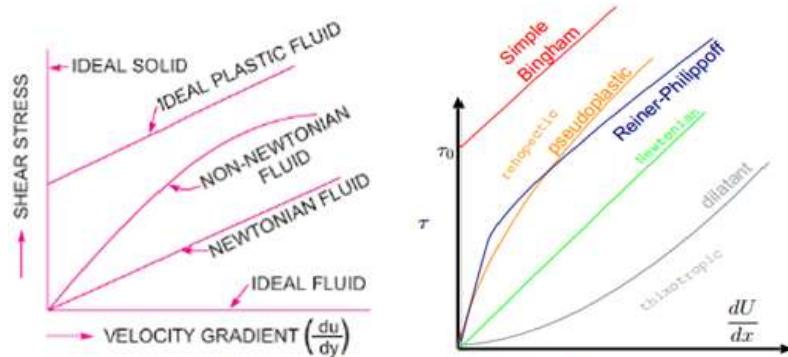


FIG. 1.4 Graphical Representation of types of fluids.

Compressibility and Bulk Modulus

Compressibility is the reciprocal of the bulk modulus of the elasticity, K which is defines as the ratio of the compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in FIG. 1.5,



Let V = Volume of gas enclosed in the cylinder

P = Pressure of gas when volume is V

Let the pressure is increased to $P + dP$, the volume of gas decreases from V or $V - dV$

Then increase in pressure = dP kgf/m²

Decrease in volume = dV

$$\therefore \text{Volumetric strain} = -\frac{dV}{V}$$

$$\therefore \text{Bulk modulus } K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}} = \frac{dP}{\frac{dV}{V}} = \frac{dP}{dV} * V$$

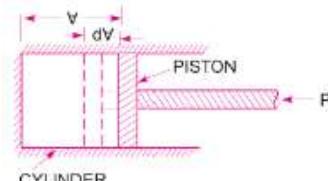


FIG. 1.5

$$\text{Compressibility} = \frac{1}{K}$$

Surface Tension

Many of us would have seen the demonstration of a needle being supported on water surface without it being wetted. This is due to the surface tension of water.



All liquids exhibit a free surface known as meniscus when in contact with vapour or gas. Liquid molecules exhibit cohesive forces binding them with each other. The molecules below the surface are generally free to move within the liquid and they move at random. When they reach the surface they reach a dead end in the sense that no molecules are present in great numbers above the surface to attract or pull them out of the surface. So they stop and return back into the liquid. A thin layer of few atomic thicknesses at the surface formed by the cohesive bond between atoms slows down and sends back the molecules reaching the surface. This cohesive bond exhibits a tensile strength for the surface layer and this is known as surface tension. Force is found necessary to stretch the surface.

Surface tension may also be defined as the work in Nm/m² or N/m required to create unit surface of the liquid. The work is actually required for pulling up the molecules with lower energy from below, to form the surface.

Another definition for surface tension is the force required to keep unit length of the surface film in equilibrium (N/m). The formation of bubbles, droplets and free jets are due to the surface tension of the liquid.



Surface Tension Effect on Solid-Liquid Interface

In liquids cohesive forces between molecules lead to surface tension. The formation of droplets is a direct effect of this phenomenon. So also the formation of a free jet, when liquid flows out of an orifice or opening like a tap. The pressure inside the droplets or jet is higher due to the surface tension.

Liquids also exhibit adhesive forces when they come in contact with other solid or liquid surfaces. At the interface this leads to the liquid surface being moved up or down forming a curved surface. When the adhesive forces are higher the contact surface is lifted up forming a concave surface. Oils, water etc. exhibit such behavior. These are said to be surface wetting. When the adhesive forces are lower, the contact surface is lowered at the interface and a convex surface results as in the case of mercury. Such liquids are called non-wetting. These are shown in **FIG. 1.6**.

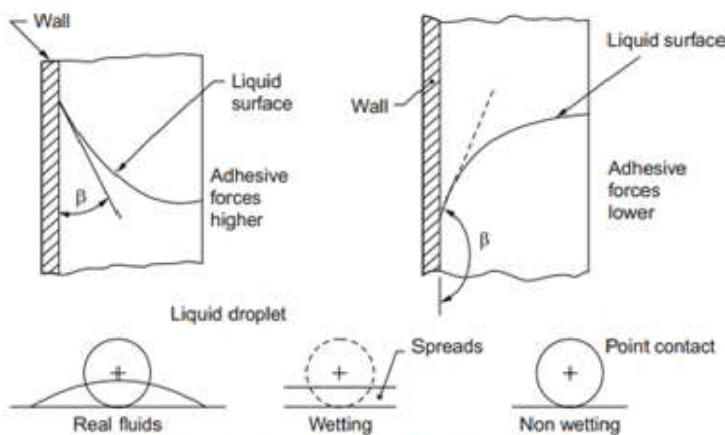


FIG. 1.6 Surface tension effect at soil-liquid interface

The angle of contact “ β ” defines the concavity or convexity of the liquid surface. It can be shown that if the surface tension at the solid liquid interface (due to adhesive forces) is σ_{s1} and if the surface tension in the liquid (due to cohesive forces) is σ_{11} then

$$\cos\beta = [(2\sigma_{s1}/\sigma_{11}) - 1]$$

At the surface this contact angle will be maintained due to molecular equilibrium. The result of this phenomenon is capillary action at the solid liquid interface. The curved surface creates a pressure differential across the free surface and causes the liquid level to be raised or lowered until static equilibrium is reached.



CAPILLARY

Capillary Rise or Depression

Let D be the diameter of the tube and β is the contact angle. The surface tension forces acting around the circumference of the tube = $\pi \times D \times \sigma$.

The vertical component of this force = $\pi \times D \times \sigma \times \cos \beta$

This is balanced by the fluid column of height, h , the specific weight of liquid being γ .

Equating, $h \times \gamma \times A = \pi \times D \times \sigma \cos \beta$, $A = \pi D^2/4$ and so

$$h = (4\pi \times D \times \sigma \times \cos \beta) / (\gamma \pi D^2) = (4\sigma \times \cos \beta) / (\rho g D)$$

This equation provides the means for calculating the capillary rise or depression. The sign of $\cos \beta$ depending on $\beta > 90^\circ$ or otherwise determines the capillary rise or depression.

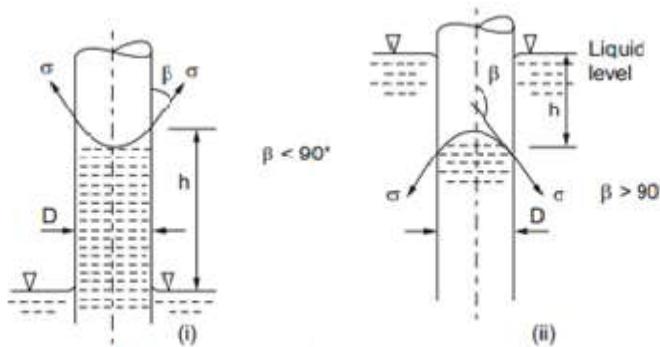


FIG. 1.7 Surface tension, (i) Capillary rise, (ii) depression

Pressure Inside a Droplet, a Free Jet and hallow bubble

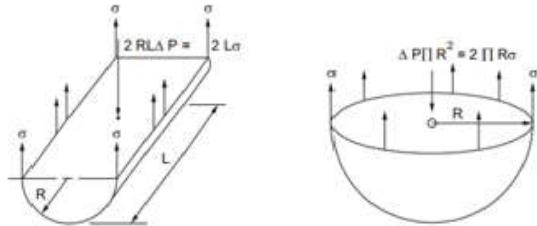


FIG 1.7 Surface tension effects on bubbles and free jets

Considering the sphere as two halves or hemispheres of diameter D and considering the equilibrium of these halves,

Pressure forces = Surface tension forces

$$(p_r - p_o)(\pi D^2/4) = \sigma \times \pi \times D$$



$$(p_i - p_o) = 4(\sigma/D) = 2(\sigma/R)$$

Considering a cylinder of length L and diameter D and considering its equilibrium, taking two halves of the cylinder.

Pressure force = $DL(p_i - p_o)$, surface tension force = $2\sigma L$

$$(p_i - p_o) = 2(\sigma/D) = (\sigma/R)$$

A hallow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$(p_i - p_o) = 4(\sigma/R)$$

Vapour Pressure and Cavitation

A change front the liquid state to the gaseous state is known as vaporization. The vaporization (which depends upon the prevailing pressure and temperature condition)

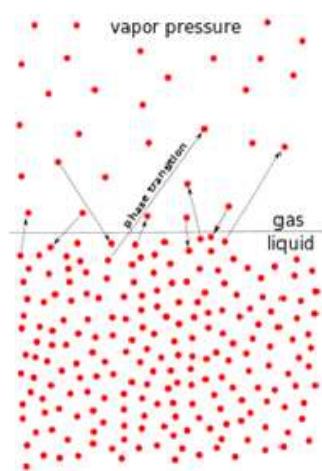


FIG 1.8 Vapour Pressure

occurs because of continuous escaping of the molecules through the free liquid surface. Consider a liquid (say water) which is confined in a closed vessel. Let the temperature of liquid is 20°C and pressure is atmospheric. This liquid will vaporize at 100°C . When vaporization takes place the molecules escapes from the free surface of the liquid. These vapour molecules get accumulated in the space between the free liquid surface and top of the vessel. These accumulated vapours exert a pressure on the liquid surface. This pressure is known as vapour pressure of the liquid or this is the pressure at which the liquid is converted into vapours. Again consider the same liquid at 20°C at



atmospheric pressure in the closed vessel. If the pressure above the liquid surface is reduced by some means the boiling temperature will also reduce. If the pressure is reduced to such an extent that it becomes equal to or less than the vapour pressure, then boiling of the liquid will start, though the temperature of the liquid is 20°C . Thus a liquid may boil even at ordinary temperature, if the pressure above the liquid surface is reduced so as to be equal or less than the vapour pressure of the liquid at that temperature. Now consider a flowing liquid in a system. If the pressure at any point in this flowing liquid becomes equal to or less than the vapour pressure, the vaporization of the liquid starts.



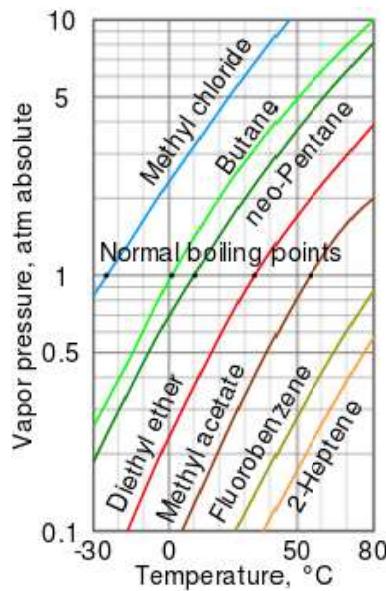


FIG 1.9 Vapour Pressures of various fluids

The bubbles of these vapours are carried by the flowing liquid into the region of high pressure where they collapse, giving rise to high impact pressure. The pressure developed by the collapsing bubbles is so high that the material from the adjoining boundaries gets eroded and cavities are formed on them. This phenomenon is known as cavitation. Hence the cavitation is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of higher pressure. When the vapour bubbles collapse, a very high pressure is created. The metallic surface, above which the liquid is flowing, is subjected to these high pressures, which cause pitting action on the surface. Thus cavities are formed on the

metallic surface and hence the name is cavitation.

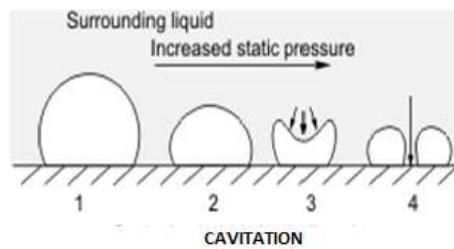


FIG 1.10 Cavitation bubble imploding close to a fixed surface generating a jet (4) of the surrounding liquids



PROBLEMS

1. Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.

Sol:

Given,

$$\text{Volume} = 1 \text{ litre} = 1/1000 \text{ m}^3 \text{ Or} = 1000 \text{ cm}^3.$$

$$\text{Weight} = 7 \text{ N}$$

- (i) Specific weight (w) = Weight / Volume = $7 / (1/1000) = 7000 \text{ N/m}^3$.
- (ii) Density (ρ) = $w/g = 7000/9.81 \text{ kg/m}^3 = 713.5 \text{ kg/m}^3$.
- (iii) Specific gravity = Density of liquid/ Density of water = $712.5/1000 = 0.7135$
[\because Density of water = 1000 kg/m^3]

2. Calculate the density, specific weight and weight of one litre petrol of specific gravity = 0.70.



Sol: Given,

$$\text{Volume} = 1 \text{ lt.} = 1 * 1000 \text{ cm}^3 = 1000/10^6 \text{ m}^3 = 0.001 \text{ m}^3$$

$$\text{Sp. Gravity} = S = 0.7$$

- (i) Density (ρ) = $S * 1000 \text{ kg/m}^3 = 0.7 * 1000 = 700 \text{ kg/m}^3$.
- (ii) Sp. Weight (w) = $\rho * g = 700 * 9.81 \text{ N/m}^3 = 6867 \text{ N/m}^3$.
- (iii) Weight (W)

$$\text{W.K.T. Sp. weight} = \text{Weight}/\text{Volume} \text{ or } w = W/V$$

$$w = W/0.001 \text{ or } 6867 = W/0.001$$

$$W = 6867 * 0.001$$

$$W = \mathbf{6.867 \text{ N.}}$$

3. If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at $y = 0$ and $y = 0.15 \text{ m}$. Take dynamic viscosity of fluid as 8.63 poises.

Sol:

$$\text{Given: } u = \frac{2}{3}y - y^2$$



$$\begin{aligned}
 \therefore \frac{du}{dy} &= \frac{2}{3} - 2y \\
 \frac{du}{dy} \text{ at } y &= 0; \\
 &= \frac{2}{3} - 2(0) = 0.667 \\
 \text{Also } \frac{du}{dy} \text{ at } y &= 0.15; \\
 &= \frac{2}{3} - 2(0.15) = 0.667 - 0.30 = 0.367
 \end{aligned}$$

Value of $\mu = 8.63$ poise = $8.63/10 \text{ Ns/m}^2$

Now shear stress is given by equation as $\tau = \mu * \frac{du}{dy}$

(i) Shear stress at $y = 0$

$$\text{i.e., } \tau \text{ at } y = 0 \rightarrow \mu * \frac{du}{dy} \text{ at } y = 0 \rightarrow 0.863 * 0.667 = \mathbf{0.5756 \text{ N/m}^2}.$$

(ii) Shear stress at $y = 0.15\text{m}$ is given by

$$\text{i.e., } \tau \text{ at } y = 0.15 \rightarrow \mu * \frac{du}{dy} \text{ at } y = 0.15 \rightarrow 0.863 * 0.367 = \mathbf{0.3167 \text{ N/m}^2}.$$

4. A plate 0.025mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.

Sol:

Given:

Distance between plates, $dy = 0.025 \text{ mm} = 0.025 * 10^{-3} \text{ m}$

Velocity of upper plate, $du = 60 \text{ cm/s} = 0.6 \text{ m/s}$

Force on upper plate, $F = 2.0 \text{ N/m}^2$.

This is the value of shear stress i.e., τ

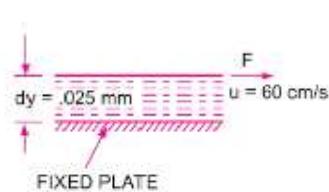
Let the fluid viscosity between the plates is μ

Using the relation $\tau = \mu * \frac{du}{dy}$

Where $du = \text{change of velocity} = u - 0 = 0.60 \text{ m/s}$

$dy = \text{change of distance} = 0.025 * 10^{-3} \text{ m}$

$\tau = \text{Force per unit area} = 2.0 \text{ N/m}^2$



$$\therefore 2.0 = \mu * [(0.60)/(0.025 * 10^{-3})]$$

$$\mu = 8.33 * 10^{-3} \text{ Ns/m}^2 \text{ or } 8.33 * 10^{-5} * 10 \text{ poise} = \mathbf{8.33 * 10^{-4} \text{ poise}}$$

Exercise

1. A flat plate of area $1.5 * 10^{-6} \text{ mm}^2$ is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.



2. Determine the intensity of the shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.

$$\text{Hint : } u = \frac{\pi D n}{60}$$

5. Calculate the dynamic viscosity of an oil, which is used for lubricating between a square plate of size 0.8 m * 0.8 m and an inclined plane with angle of inclination 30° as shown in fig. the weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. the thickness of oil film is 1.5mm

Sol: Given,

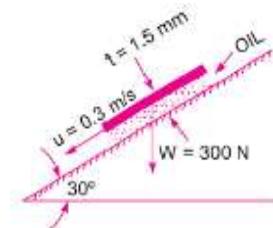
$$\text{Area of the plate } A = 0.80 * 0.80 = 0.64 \text{ m}^2.$$

$$\text{Angle of plane, } \theta = 30^\circ$$

$$\text{Weight of plate, } W = 300 \text{ N}$$

$$\text{Velocity of plate, } u = 0.30 \text{ m/s}$$

$$\text{Thickness of oil film, } t = dy = 1.5 * 10^{-3} \text{ m.}$$



Let the viscosity of fluid between plate and inclined plane is μ

$$\text{Component of weight } W, \text{ along the plane} = W \cos 60^\circ = 300 \cos 60^\circ = 150 \text{ N}$$

Thus the shear force, F , on the bottom surface of the plate = 150 N

$$\text{Shear stress} = \tau = F/\text{Area} = 150/0.60 \text{ N/m}^2.$$

$$\text{From relation, } \tau = \mu * \frac{du}{dy}$$

We have,

$$150/0.64 = \mu * [(0.30)/(1.5 * 10^{-3})]$$

$$\therefore \mu = 1.17 \text{ N s/m}^2 = 11.7 \text{ poise}$$



Exercise

1. Two horizontal plates are placed 1.25 cm apart, the space between them being filled with oil viscosity 14 poise. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.

Ans: 280 N/m²

2. The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. the thickness of the oil film is 12.5 mm. the upper plate, which is moves at 2.5 m per sec requires a force of 98.1 N to maintain the speed. Determine:

(i) The dynamic viscosity of the oil in poise, and

(ii) The kinematic viscosity of the oil in stokes if the sp. Gravity of the oil is 0.95.

Ans: 13.635 poise, 14.35 stokes.

3. Find the kinematic viscosity of an oil having density 981 kg/m³. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at the point is 0.2 per second.

Ans: 12.5 stokes



4. Determine the sp. Gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes.

Ans: 1.43

5. The velocity distribution for flow over a flat plate is given by $u = \frac{3}{4}y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate. Determine the shear stress at $y = 0.15$ m. Take dynamic viscosity of fluid as 8.6 poise.

Ans : 0.3825 N/m².

6. The dynamic viscosity of an oil, used for lubricating between a shaft and the sleeve is 6 poise. The shaft is of diameter 0.04m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm the thickness of the oil film is 1.5mm.

Sol: Given,

$$\text{Viscosity } \mu = 6 \text{ poise} = 0.6 \text{ N s/m}^2.$$

$$\text{Dia. Of shaft } D = 0.4 \text{ m}$$

$$\text{Speed of shaft, } N = 190 \text{ rpm}$$

$$\text{Thickness of oil film, } t = 1.5 \text{ mm} = 1.5 * 10^{-3} \text{ m.}$$

$$\text{Sleeve length, } L = 90 \text{ mm} = 90 * 10^{-3} \text{ m}$$

$$\text{Tangential velocity of shaft, } u = \frac{\pi D n}{60} = \frac{\pi * 0.4 * 190}{60} = 3.98 \text{ m/s}$$

$$\text{From relation } \tau = \mu * \frac{du}{dy}$$

$$\text{Where, } du = u - 0 = 3.98 \text{ m/s}$$

$$dy = t = 1.5 * 10^{-3} \text{ m}$$

$$\therefore \tau = 10 * (3.98 / 1.5 * 10^{-3}) = 1592 \text{ N/m}^2.$$

This is shear stress on the shaft.

∴ Shear force on the shaft, $F = \text{Shear stress} * \text{area}$

$$= 1592 * \pi D * L = 1592 * \pi * 0.40 * 90 * 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} * (D/2) = 180.05 * (0.4/2) = 36.01 \text{ Nm}$

$$\therefore \text{Power lost} = \frac{2\pi NT}{60} = \frac{2\pi * 190 * 36.01}{60} = 716.48 \text{ W.}$$

7. If the velocity profile of a fluid over a plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.

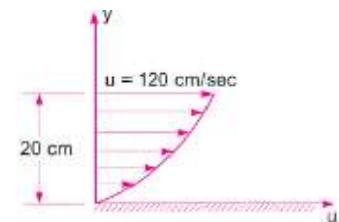
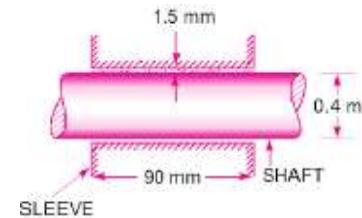
Sol. Distance of vortex from plate = 20cm

$$\text{Velocity at vertex, } u = 120 \text{ cm/sec}$$

$$\text{Viscosity, } \mu = 8.5 \text{ poise} = 0.85 \text{ Ns/m}^2.$$

The velocity profile is given parabolic and equation of velocity profile is

$$U = ay^2 + by + c$$



Where a, b and c are constants. Their values are determined from boundary conditions as:

- (a) At $y = 0$, $u = 0$
- (b) At $y = 20$ cm, $u = 120$ cm/s
- (c) At $y = 20$ cm, $\frac{du}{dy} = 0$.

Substituting boundary condition (a) in equation (i), we get

$$C = 0$$

Boundary condition (b) on substitution in (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b$$

Boundary condition (c) on substitute in equation (i) gives

$$\frac{du}{dy} = 2ay + b$$

$$0 = 2 * a * 20 + b = 400a + 20b$$

Solving equation (ii) and (iii) for a and b

$$\text{From equation (iii), } b = -40a$$

Substitute this value in equation (ii), we get

$$120 = 400a + 20 * (-40a) = 400a - 800a = -400a$$

$$\therefore a = (120/-400) = (-3/10) = -0.3$$

$$\therefore b = -40 * (-0.3) = 12.0$$

Substituting this value of a, b and c in equation (i),

$$u = -0.3y^2 + 12y$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 * 2y + 12 = -0.6y + 12$$

$$\text{at } y = 0, \text{ velocity gradient, } \frac{du}{dy} = -0.6 * 0 + 12 = 12/\text{s.}$$

$$\text{at } y = 10 \text{ cm, } \frac{du}{dy} = -0.6 * 10 + 12 = -6 + 12 = 6/\text{s.}$$

$$\text{at } y = 20 \text{ cm, } \frac{du}{dy} = -0.6 * 20 + 12 = -6 + 12 = 0/\text{s.}$$



Shear Stress

Shear stress is given by, $\tau = \mu * \frac{du}{dy}$

$$(i) \text{ Shear stress at } y = 0, \tau = \mu * \frac{du}{dy} \text{ at } y = 0 \rightarrow 0.85 * 12.0 = 10.2 \text{ N/m}^2.$$

$$(ii) \text{ Shear stress at } y = 10, \tau = \mu * \frac{du}{dy} \text{ at } y = 10 \rightarrow 0.85 * 6.0 = 5.10 \text{ N/m}^2.$$

$$(iii) \text{ Shear stress at } y = 20, \tau = \mu * \frac{du}{dy} \text{ at } y = 20 \rightarrow 0.85 * 0 = 0.$$

8. Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerin. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surface at a speed of 0.6 m/s, if:

(i) The thin plate is in the middle of the two plane surfaces, and



(ii) The thin plate is at a distance of 0.8 cm from one of the plane surfaces? Take the dynamic viscosity of glycerin = $8.10 \times 10^{-1} \text{ Ns/m}^2$.

Sol: Given,

Distance between two large surfaces = 2.4 cm

Area of this plate, $A = 0.5 \text{ m}^2$.

Velocity of thin plate, $u = 0.6 \text{ m/s}$.

Viscosity of glycerin, $\mu = 8.10 \times 10^{-1} \text{ Ns/m}^2$

CASE - I: When the thin plate is in the middle of the two plane surfaces.

Let F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate.

Then, $F = F_1 + F_2$

The shear stress (τ_1) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu * \left[\frac{du}{dy} \right]_1$$

Where, du = relative velocity between thin plate and upper large plane surface = 0.6 m/sec

dy = distance between thin plate and upper large plane surface = 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

$$\therefore \tau_1 = 8.10 \times 10^{-1} * (0.6/0.012) = 40.5 \text{ N/m}^2$$

Now shear force, F_1 = shear stress * area

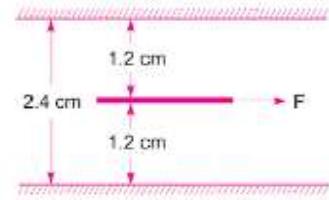
$$= \tau_1 * A = 40.5 * 0.50 = 20.25 \text{ N}$$

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\begin{aligned} \tau_2 &= \mu * \left[\frac{du}{dy} \right]_2 \\ &= 8.10 \times 10^{-1} * (0.6/0.012) = 40.5 \text{ N/m}^2 \end{aligned}$$

$$\therefore \text{Shear force, } F_2 = \tau_2 * A = 40.5 * 0.5 = 20.25 \text{ N}$$

$$\therefore \text{Total shear force } F = F_1 + F_2 = 20.25 + 20.25 = 40.5 \text{ N}$$



CASE II: When the thin plate is at a distance of 0.8cm from one of the plane surface.

Let the thin plate is at a distance 0.80 cm from the lower plane surface.

Then distance of the plate from the upper plane surface = $2.4 - 0.8 = 1.6 \text{ cm} = 0.016 \text{ m}$

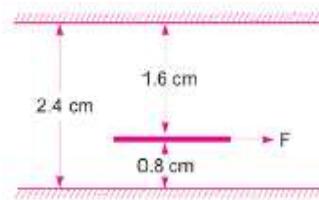
The shear force on the upper side of the thin plate,

$$\begin{aligned} F_1 &= \text{shear stress * area} = \tau_1 * A = \mu * \left[\frac{du}{dy} \right]_1 * A \\ &= 8.10 \times 10^{-1} * (0.6/0.016) * 0.5 = 15.18 \text{ N} \end{aligned}$$

The shear force on the lower side of the thin plate,

$$\begin{aligned} F_2 &= \tau_2 * A = \mu * \left[\frac{du}{dy} \right]_2 * A \\ &= 8.10 \times 10^{-1} * (0.6/(0.8/100)) * 0.5 = 30.36 \text{ N} \end{aligned}$$

$$\therefore \text{Total force required} = F_1 + F_2 = 15.18 + 30.36 = 45.54 \text{ N}$$



Exercise

1. A Newtonian fluid is filled in the clearance between a shaft and a concentric sleeve. The sleeve attains a speed of 50 cm/s, when a force of 40 N is applied to the sleeve parallel to the shaft. Determine the speed if a force of 200N is applied.

Ans: 250 cm/s

2. A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 rpm. Determine the viscosity of the fluid.

Ans: 8.64 poise.

3. A vertical gap 2.2 cm wide of infinite extent contains a fluid of viscosity 2.0 Ns/m^2 and sp. Gravity 0.9. A metallic plate $1.2 \text{ m} \times 1.2 \text{ m} \times 0.2 \text{ cm}$ is to be lifted up with a constant velocity of 0.15 m/sec, through the gap. If the plate is in the middle of the gap, filled the force required. The weight of the plate is 40 N.

Ans: 100.97 N

4. Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions $\sigma = 0.0725 \text{ N/m}$ for water and $\sigma = 0.52 \text{ N/m}$ for mercury in contact with air, the sp. Gravity for mercury is given as 13.6 and angle of contact = 130° .

Ans: $h = -0.40 \text{ cm}$

