UNIT - II

FLUID STATICS

CONTENTS

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FLUID STATICS

Pressure at a point – Absolute and gauge pressures – Pascal's and Hydrostatic laws – Pressure measurement – Manometers and mechanical gauges – Hydrostatic thrust on plane and curved surfaces – Buoyancy and flotation –Metacentric height.

Objectives:
When you finish reading this chapter, you should be able to

- Determine the variation of pressure in a fluid at rest
- Calculate the forces exerted by a fluid at rest on plane or curved submerged surfaces
- Determine the buoyancy, flotation and Metacentric height of the objects.

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2.1 PRESSURE

Pressure is defined as a normal force exerted by a fluid per unit area. We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is normal stress. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m2), which is called a **pascal**(Pa). That is, the pressure unit pascal is too small for pressures encountered in practice. Therefore, its multiples kilopascal (1 kPa = 103 Pa) and megapascal(1 MPa = 106 Pa) are commonly used. Three other pressure units commonly used in practice are bar, standard atmosphere, and kilogram-force per square centimeter:

1bar = 10^5 Pa = 0.1 MPa = 100 kPa

1 atm = 101,325 Pa = 101.325 kPa = 1.101325 bars

1 kgf/cm^2 = 9.807 N/cm^2 = 9.807 * 104 N/m^2

= 9.807 * 10^4 Pa = 0.9807 Bar= 0.9679 atm

Note: The pressure units bar, atm, and kgf/cm^2 are almost equivalent to each other. In the English system, the pressure unit is pound-force per square inch (lbf/in^2, or psi), and 1 atm = 14.696 psi. The pressure unitskgf/cm2 and lbf/in^2 are also denoted by kg/cm^2 and lb/in^2, respectively, and they are commonly used in tire gages. It can be shown that 1 kgf/cm^2 = 14.223 psi.

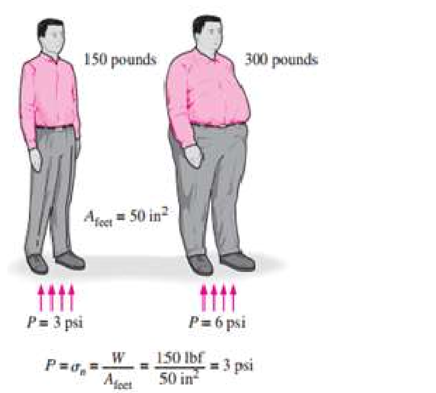


FIG 3.1 The normal stress or pressure on the feet of a chubby person is much greater than on the feet of much greater than on the feet of slim person

Pressure is also used for solids as synonymous to normal stress, which is force acting perpendicular to the surface per unit area. ***For example***, a 150-pound person with a total foot imprint area of 50 in^2 exerts a pressure of 150 lbf/50 in^2 = 3.0 psi on the floor **FIG 3.1.**If the person stands on one foot, the pressure doubles. If the person gains excessive weight, he or she is likely to encounter foot discomfort because of the increased pressure on the foot (the size of the foot does not change with weight gain). This also explains how a person can walk on fresh snow without sinking by wearing large snowshoes, and how a person cuts with little effort when using a sharp knife.

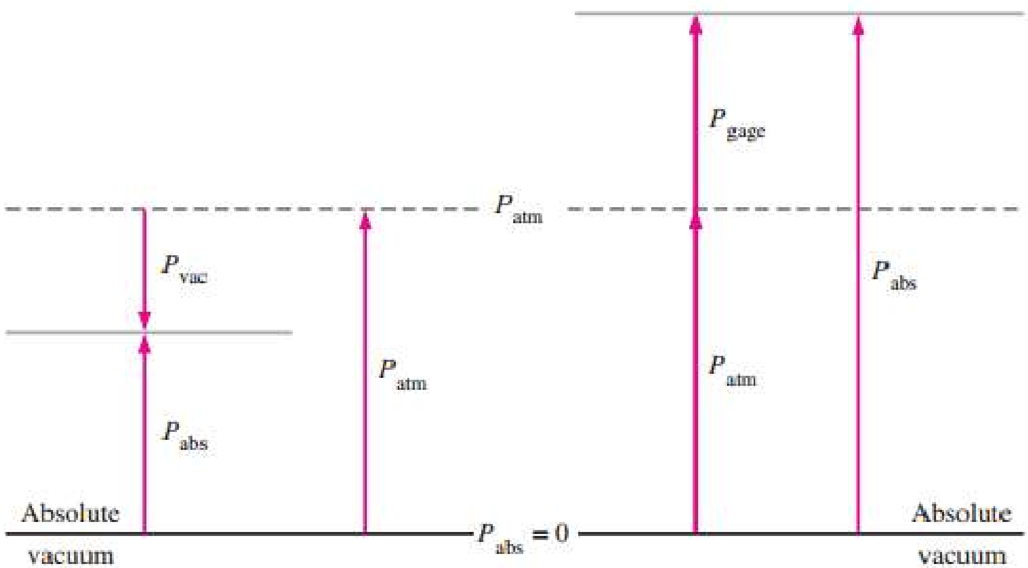
3.1.1 ABSOLUTE, GAGE, ATMOSPHERIC AND VACCUM PRESSURES

FIG 3.2 Pressure Gauges

The actual pressure at a given position is called the **absolute pressure,** and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, are calibrated to read zero in the atmosphere FIG 3.2, and so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the **gage pressure.** Pressures below atmospheric pressure are called **vacuum pressures** and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure. Absolute, gage, and vacuum pressures are all positive quantities and are related to each other by

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$



3.2PRESSURE AT A POINT – PASCAL'S LAW

It states that "***the pressure or intensity of pressure at a point in a static fluid is equal in all directions.***"

This is proved as:

The fluid element is of very small dimensions i.e., dx, dy and ds.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in **FIG. 3.4**. Let the width of the element perpendicular to the plate of paper is unity and p_x, p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let ∠ABC = θ. Then the forces acting on the element are:

1. Pressure forces normal to the surfaces, and
2. Weight of element in the vertical direction.

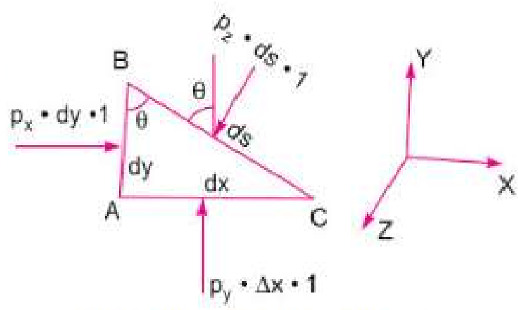


FIG 3.4 Forces on a fluid element

The forces on the faces are:

Force on the face AB = p_x* Area of face AB

= p_x * dx * 1

Similarly, force on the face AC = p_y * dy * 1

Force on the face AC = p_z * ds * 1

Weight of element = (Mass of element) x g

= (Volume * ρ) * g = $\left(\frac{AB*AC}{2}\right)$* 1*ρ*g

Where ρ = density of fluid

Resolving the forces in x- direction, we have

= p_x * dy * 1 – p (ds*1) sin (90°-θ) orp_x * dy * 1 – p_z (ds*1) cos θ = 0.

But from fig., ds cosθ = AB = dy

∴p_x * dy * 1 – p_z dy = 0.

$$\therefore \boldsymbol{p_x = p_z}$$

Similarly, resolving the forces in y-direction, we get

= p_y * dx * 1 – p_z (ds*1) cos (90°-θ) -$\frac{dx*dy}{2}$*1*ρ*g

= p_y * dx - p_z * ds sinθ - $\frac{dx*dy}{2}$*ρ*g = 0

But from fig., ds sinθ = dx and also the element is very small and hence weight is negligible.

∴p_y * dx − p_z dx = 0.

$$\therefore \boldsymbol{p_y = p_z}$$

The above equation shows that the pressure at any point x,y and z directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

3.3 PRESSURE VARIATION IN A FLUID AT REST – HYDROSTATIC LAW

The pressure at any point in a fluid at rest is obtained by the Hydrostatic law which states that ***the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.*** This is proved as:

Consider a small fluid element as shown in **FIG. 3.5**

Let ΔA = Cross-sectional area of the element

ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

The forces acting on the fluid element are:

1. Pressure force acting on AB = p * ΔA and acting perpendicular to face AB in the downward direction.
2. Pressure force on CD = $\left(p + \frac{\partial p}{\partial z}\Delta Z\right)$ * ΔA, acting perpendicular to face CD, vertically upward direction.
3. Weight of fluid element = Density * g * volume = ρ * g * (ΔA * ΔZ).
4. Pressure forces on surface BC and AD are equal and opposite. For equilibrium of fluid element, we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial z}\Delta Z\right) * \Delta A + \rho * g * (\Delta A * \Delta Z) = 0$$

$$p\,\Delta A - p\,\Delta A - \frac{\partial p}{\partial z}\Delta Z * \Delta A + \rho * g * (\Delta A * \Delta Z) = 0$$

$$-\frac{\partial p}{\partial z}\Delta Z * \Delta A + \rho * g * (\Delta A * \Delta Z) = 0$$

$$\frac{\partial p}{\partial z}\Delta Z * \Delta A = \rho * g * (\Delta A * \Delta Z)$$

$$\frac{\partial p}{\partial z} = \rho * g$$

$$\therefore \frac{\partial p}{\partial z} = \rho * g = w \qquad \text{--------- eq 3.1}$$

Where, w = weight density of fluid.

Equation 3.1 states that the rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point. This is **Hydrostatic Law**.

By integrating the above equation 3.1 for liquids, we get

$\int dp = \int \rho * g * dZ$

P = ρ*g*Z --------------- eq. 3.2

Where p is pressure above the atmospheric pressure and Z is the height of the point from free surfaces.

From equation 3.2, we have $Z = \frac{p}{\rho * g}$ --------------- eq. 3.3

Here Z is called **pressure head**.

3.4 Measurement of pressure

The pressure of a fluid is measured by the following devices:

1. Manometers
2. Mechanical Gauges.

3.4.1 Manomters

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as:

(a) Simple manometers
(b) Differential manometers

3.4.2. Mechanical Gauges: Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are:

(a) Diaphragm pressure gauges, (b) Bourdon tube pressure gauge,

(c) Dead- weight pressure gauge, (d) Bellows Pressure gauge.

3.5. SIMPLE MANOMTERS

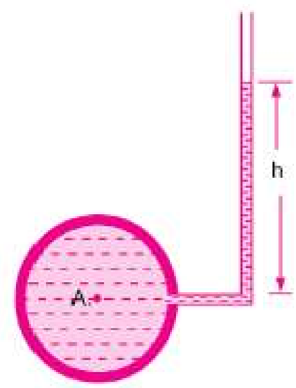
A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are:

1. Piezometer;
2. U-tube Manometer, and
3. Single Column Manometer.

3.5.1. Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in **FIG.3.6**. The rise of liquid gives the pressure head at that point. If at a point **A**, the height of liquid say water is ***h*** in piezometer tube, then pressure at A ,

$$= \rho * g * h \ N/m^2.$$

3.5.2. U-tube Manometer.



It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in **FIG. 3.7**. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

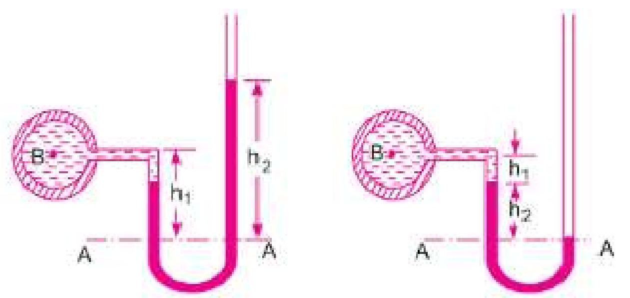
(a) For Gauge Pressure:Let B is the point at which pressure is to be measured, whose values is p.

The datum line is A – A.

Let h_1 = Height of light liquid above the datum line

h_2 =Height of heavy liquid above the datum line

S_1 = Sp. Gr. Of light liquid



(a) For gauge pressure (b) For vacuum pressure

S_2 = Sp.gr of heavy liquid

ρ_1 = Density of the light liquid = 1000 * S_1

ρ_2 = Density of the heavy liquid = 1000 * S_2

As the pressure is the same for the horizontal surface. Hence pressures above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column = $p + \rho_1 * g * h_1$

Pressure above A-A in the right column = $\rho_2 * g * h_2$

Hence equating the two pressures, $p + \rho_1 * g * h_1 = \rho_2 * g * h_2$

$\therefore$ **$p = (\rho_2 * g * h_2 - \rho_1 * g * h_1)$**

(b) **For Vacuum Pressure:** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in FIG 3.7 (b). then

Pressure above A-A in the left column = $\rho_2 * g * h_2 + \rho_1 * g * h_1 + p$

Pressure head in the right column above A-A = 0

$\therefore$ $\rho_2 * g * h_2 + \rho_1 * g * h_1 + p = 0$

$\therefore$ **$p = -(\rho_2 * g * h_2 + \rho_1 * g * h_1)$**

3.5.3 **Single Column Manometer.** Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in **FIG. 3.8**. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

1. Vertical Single Column Manometer.

2. Inclined Single Column Manometer.

1. Vertical Single Column Manometer:FIG 3.9 shows the vertical single column manometer. Let X-X is the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

h_2 = rise of heavy liquid in right limb

h_1 = height of Centre of pipe above X-X

p_A = pressure at A, which is to be measured

A = Cross – sectional area of the reservoir

a= Cross-sectional area of the right limb

S_1 = Sp.gr. of liquid in pipe

S_2 = Sp.gr. of heavy liquid in reservoir and right limb

ρ_1 = density of liquid in pipe

ρ_2 = density of the liquid in the reservoir

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb

$\therefore A * \Delta h = a * h_2$

$\therefore \Delta h = \frac{a * h_2}{A}$

Now consider the datum line Y-Y as shown in Fig. Then the pressure in the right limb above Y-Y

$= \rho_2 * g * (\Delta h + h_2)$

Pressure in the left limb above Y-Y = $\rho_1 * g * (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 * g * (\Delta h + h_2) = \rho_1 * g * (\Delta h + h_1) + p_A$$

$$p_A = \rho_2 * g * (\Delta h + h_2) - \rho_1 * g * (\Delta h + h_1)$$

$$= \Delta h(\rho_2 * g - \rho_2 * g) + h_2 * \rho_2 * g - h_1 * \rho_1 * g$$

But ,$\Delta h = \frac{a * h_2}{A}$

$\therefore$**$p_A = \frac{a*h_2}{A} * (\rho_2*g - \rho_2*g) + h_2*\rho_2*g - h_1*\rho_1*g$**

As the area A is very large as compared to a, hence ratio a/A becomes very small and can be neglected.

Then, **$p_A = h_2*\rho_2*g - h_1*\rho_1*g$**

From above equation, it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

2. Inclined Single Column Manometer: In FIG 3.10 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

Let L = length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = vertical rise of heavy liquid in right limb from X-X = $L*\sin\theta$

From relation, we have

$p_A = h_2*\rho_2*g - h_1*\rho_1*g$

Substituting the value of h_2, we get

$p_A = \sin\theta*\rho_2*g - h_1*\rho_1*g$

3.6 DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing heavy liquid, whose two ends are connected to the points, whose differences of pressure is to be measured. Most commonly types of differential manometers are:

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

3.6.1 U-tube differential manometer: FIG 3.11shows the differential manometers of U-tube type.

In fig 3.11(a), the two point A and B are at different level and also contain liquids of different sp.gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are p_A and p_B.

Let h = differential of mercury level in the U-tube

y = distance of the Centre of B, from the mercury level in the right limb.

x = distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_x = Density of heavy liquid or mercury.

Taking datum line at X-X,

Pressure above X-X- in the left limb = $\rho_1*g*(h+x)+p_A$

Wherep_A = pressure at A

Pressure above X-X in the right limb = $\rho_g*g*h* + \rho_2*g*y+p_B$

Wherep_B = pressure at B

Equating the two pressures, we have

$$\rho_1*g*(h+x)+p_A = \rho_g*g*h + \rho_2*g*y+p_B$$

$$p_A – p_B= \rho_g*g*h + \rho_2*g*y - \rho_1*g*(h+x)$$

$$= h*g*(\rho_g-\rho_1) + \rho_2*g*y - \rho_1*g*x$$

∴ Difference of Pressure at A and B are at the same level and contains the same liquid of density ρ_1, then

Pressure above X-X in right limb = $\rho_g*g*h + \rho_1*g*x+p_B$

Pressure above X-X in left limb = $\rho_1*g*(h+x)+p_A$

Equating the two pressure

$$\rho_g*g*h + \rho_1*g*x+p_B = \rho_1*g*(h+x)+p_A$$

$$\therefore \quad p_A - p_B = \rho_g*g*h + \rho_1*g*x - \rho_1*g*(h+x)$$

$$\therefore \quad \mathbf{p_A - p_B = g*h(\rho_g - \rho_1)}$$

3.6.2. Inverted U-tube Differential Manometer. It consists of an inverted U-tube containing a light liquid. The two ends of the tube are connected to the pints whose difference of pressure is to be measured. It is used for measuring difference of low pressures. FIG 3.11 shows an inverted U-tube manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let h_1= Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = difference of light liquid

ρ_1 = density of liquid A

ρ_2 = density of liquid B

ρ_s = density of light liquid

p_A = Pressure at A

p_B = pressure at B

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1*g*h_1$$

Pressure in the right limb below X-X

$$= p_B - \rho_2*g*h_2-\rho_s*g*h$$

Equating the two pressure

$p_A - \rho_1 * g * h_1 = p_B - \rho_2 * g * h_2 - \rho_s * g * h$

$\mathbf{p_A - p_B = \rho_1 * g * h_1 - \rho_2 * g * h_2 - \rho_s * g * h}$

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PROBLEMS
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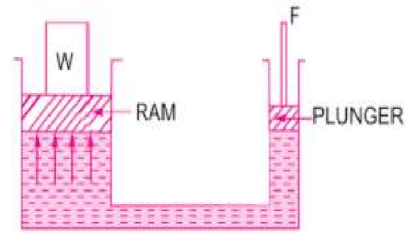
1. A hydraulic press has a ram of 30cm diameter and a plunger of 4.5cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.

Sol: **Given:**



Dia. of ram, D = 30 cm = 0.30m

Dia. of plunger, d = 4.5 cm = 0.045 m

Force on plunger, F = 500 N

Weight lifted = W

Area of the ram, A = $\frac{\pi}{4} * D^2 = \frac{\pi}{4} * 0.3^2$ = 0.07068 sq.m.

Area of the plunger, a = $\frac{\pi}{4} * d^2 = \frac{\pi}{4} * 0.045^2$ = 0.00159 sq.m.

Pressure intensity due to plunger = $\frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{500}{0.00159}$ N/sq.m.

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$$= \frac{500}{0.00159} = 314465.4 \text{ N/sq.m.}$$

But pressure intensity at ram = $\frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{0.07068}$ N/sq.m.

$$\frac{W}{0.07068} = 314465.4$$

∴ Weight = 314465.4 * 0.07068 = 22222 N = **22.22 kN**

2. Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. Gr. 0.8, and (c) mercury of sp. Gr. 13.6. Take density of water, ρ = 1000 kg/m³.

Sol: Given,

Height of liquid column, Z = 0.3 m.

The pressure at any point in a liquid is given by equation as p = ρ*g*Z

(a) For water, ρ = 1000 kg/cu.m.

p = ρ*g*Z = 1000 * 9.81 * 0.3 = 2943 N/sq.m = 0.2943 N/sq.cm

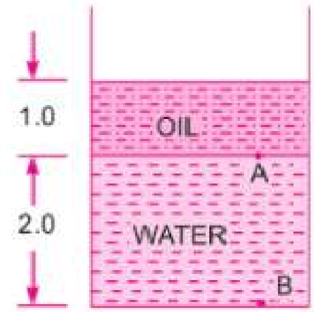
(b) For oil of sp. Gr. 0.8,

p = 0.2354 N/sq.cm.

(c) For mercury, sp. gr. = 13.6

p = 4.002 N/sq.cm.

3. An open tank contains water up to a depth of 2m and above it an oil of sp.gr. 0.9 for a depth of 1m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.



Sol: Given,

Height of water, Z_1 = 2m

Height of the oil, Z_2 = 1 m

Sp.gr. of oil, S_o = 0.9

Density of the water, ρ_1 = 1000 kg/m^3.

Density of the oil, ρ_2 = Sp.gr. of oil * Density of water

= 0.9*1000 = 900 kg/m^3.

Pressure intensity at any point is given by

$$p = \rho * g * Z$$

(i) At interface at A, $p_A = \rho_2 * g * Z_2$ = 900*9.81*1.0 = 0.8829 N/sq.cm

(ii) At the bottom at B, $p_B = \rho_2 * g * Z_2 + \rho_1 * g * Z_1$ = (900*9.81*1)+(1000*9.81*2.0)

= 2.8449 N/sq.cm

4. Express pressure intensity of 7.5 kg(f)/cm^2. In all pressure units. Take the barometer reading as 76 cm of mercury.

Sol:

(A) GAUGE UNITS

(a) p = 7.5 kg(f)/cm^2.

(b) p = 7.5 *10^{4}kg(f)/m^2.

(c) h = $\frac{p}{w} = \frac{7.5*10^4}{1000}$ = 75 m of water

(d) h = $\frac{p}{w} = \frac{7.5*10^4}{13.6*1000}$ = 5.51 m of mercury.

(e) p = 9.810*75 = 73.575*10^4 N/m^2.

(B) ABSOULTE UNITS

Absolute pressure = Gauge pressure + Atmospheric pressure

Atmospheric pressure = 76 cm of mercury

= $\frac{76*13.6}{100}$ = 10.34 m of water

$= \frac{76*13.6*1000}{100} = 1.034 * 10^4 kg(f)/m^2$.

$= \frac{76*13.6*1000}{100*10^4} = 1.034\ kg(f)/cm^2$.

$= \frac{76*13.6*9810}{100} = 10.14 * 10^4\ N/m^2$.

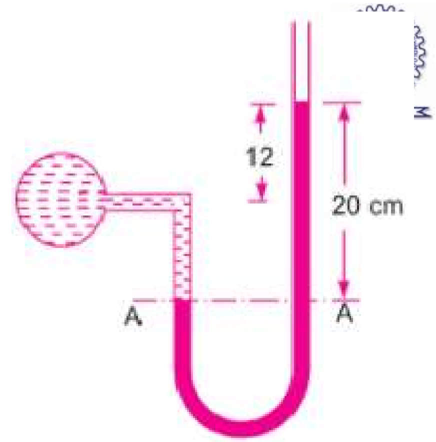
(a) Absolute pressure = (7.5 + 1.034) = 8.534 $kg(f)/cm^2$.
(b) Absolute pressure = $(7.5*10^4 + 1.034*10^4) = 8.534 * 10^4 kg(f)/m^2$.
(c) Absolute pressure head = (75+ 10.34) = 85.34 m of water
(d) Absolute pressure head = (5.51 + 0.76) = 6.27 m of mercury
(e) Absolute pressure = 6.27/0.76 = 8.25 atmospheres.
(f) Absolute pressure = $(73.58*10^4 + 10.14*10^4) = 83.72 * 10^4\ N/m^2$.

SIMPLE MANOMETER

5. **The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp.gr. 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.**

Sol: Given,

Sp. gr. of fluid, $S_1 = 0.90$



∴ Density of fluid, $\rho_1 = S_1 * 1000 = 0.90 * 1000 = 900\ kg/m^3$.

Sp.gr. of mercury, $S_2 = 13.6$

∴ Density of mercury, $\rho_2 = 13.6 * 1000\ kg/m^3$.

Difference of mercury level, h_2 = 20 cm = 0.2 m

Height of fluid from A-A, h_1 = 20 -12 = 8cm = 0.08m

Let p = pressure of fluid in pipe

Equating the pressure above A-A, we get

$P + \rho_1 g h_1 = \rho_2 g h_2$

p+ (900*9.81*0.08) = 13.6*1000*9.81*0.20

p = 13.6*1000*9.81*0.20-900*9.81*0.08

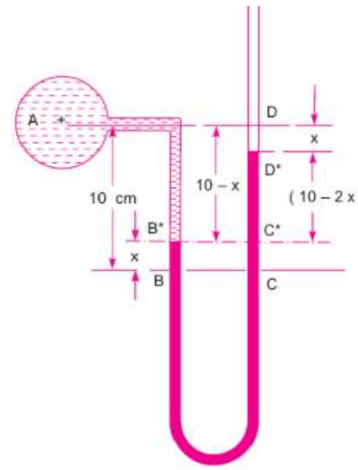
p = 2.597 N/cm^2.

6. **A U-tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury**

and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m^2, calculate the new difference in the level of mercury. Sketch the arrangements in both cases.

Sol: Given:

Difference of mercury = 10 cm = 0.1 m



The arrangement is shown in fig (a)

CASE-I

Let p_A = Pressure of water pipe line at point A

The points B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C. But pressure at B

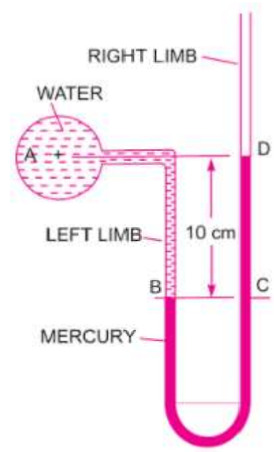
= Pressure at A + Pressure due to 10 cm of water

= $p_A + \rho*g*h$ = p_A+ (1000*9.81*0.1)

= **p_A + 981 N/m^2.**

Pressure at C = Pressure at D + Pressure due to 10 cm of mercury

= $0 + \rho_o*g*h_o$



Where ρ_o = 13.6*1000 kg/m^3 and h_o = 10 cm = 0.1m

∴ Pressure at C = 0 +(13.6*1000)*9.81*0.1

But pressure at B is equal to pressure at C. hence equating the equation (i) and (ii), we get

∴p_A+ 981 = 13341.6

p_A = 12360.6 N/m^2.

CASE-II

Given p_A= 9810 N/m^2.

Find new difference of mercury level. The arrangement is shown in Fig (b). In this case the pressure at A is 9810 N/m^2 which is less than 12360.6 N/m^2. Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall mercury in right limb as the total volume of mercury remains same.

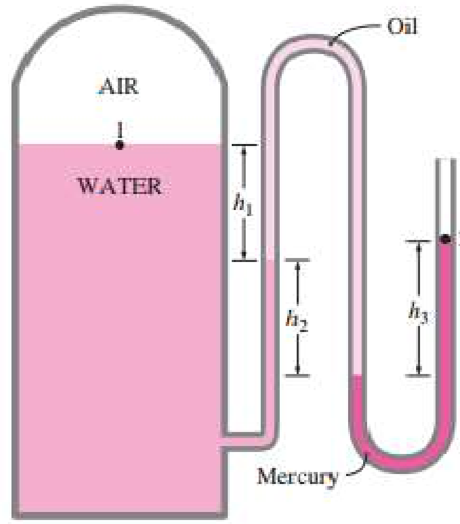
Let x = raise of mercury in left limb in cm

Then fall of mercury in right limb = x cm

The points B, C and D show the initial conditions whereas point B*, C* and D* show the final conditions.

The pressure at B* = pressure at C*

Pressure at A + Pressure due to (10-x) cm of water = Pressure at D* + pressure due to (10-2x) cm of mercury



$p_A + \rho_1 * g * h_1 = p_D* + \rho_2 * g * h_2$

$1910 + 1000 * 9.81 * \frac{10-x}{100} = 0 + (13.6*1000)*9.81*\frac{10-2x}{100}$

Dividing by 9.81, we get

1000+100 – 10x = 1360 – 272x

x = 0.992 cm

∴ New difference of mercury = 10 -2x = 10 – 2*0.992 = 8.016 cm.

7. **The water in a tank is pressurized by air, and the pressure is measured by a multi-fluid manometer as shown in Fig. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if h_1 = $0._1$ m, h_2 = 0.2 m, and h_3 = 0.35 m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.**

Sol: The pressure in a pressurized water tank is measured by a multi-fluid manometer. The air pressure in the tank is to be determined.

Assumption The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air-water interface.

Properties The densities of water, oil, and mercury are given to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

Analysis Starting with the pressure at point 1 at the air - water interface, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives

$P_1 + \rho_{water} * g * h_1 + \rho_{oil} * g * h_2 - \rho_{mercury} * g * h_3 = P_{atm}$

Solving for P_1 and substituting

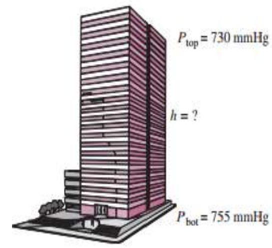
$$P_1 = P_{atm} - \rho_{water}*g*h_1 - \rho_{oil}*g*h_2 + \rho_{mercury}*g*h_3$$

$$= P_{atm} + g*(\rho_{mercuryr}*h_3 - \rho_{water}*h_1 + \rho_{oil}*h_2)$$

$$= 85.6\ kPa + (9.81\ m/s^2)[(136000\ kg/m^3)*(0.35\ m) - (1000\ kg/m^3)*(0.1\ m) - (850\ kg/m^3)*(0.2\ m)]\left(\frac{1\ N}{1\ kg.m/s^2}\right)\left(\frac{1\ kPa}{1000\ N/m^2}\right)$$

P_1 = 130 kPa.

Exercise – II (Basic pressure measurement & Simple Manometers)



1. **The basic barometer can be used to measure the height of a building. If the barometric readings at the top and at the bottom of a building are 730 and 755 mmHg, respectively, determine the height of the building. Assume an average air density of 1.18 kg/m^3.**

2. **A hydraulic press has a ram of 20cm diameter and a plunger of 3cm diameter. It is used for lifting a weight of 30kN. Find the force required at the plunger.**
   Ans. F = 675.2N

3. **The pressure intensity at a point in a fluid is given 3.924 N/cm^2. Find the corresponding height of fluid when the fluid is: (a) water, and (b) oil sp.gr. 0.90**
   Ans. (a) 4m of water, and (b) 4.44m of oil

4. **An oil of sp.gr. 0.9 is contained in a vessel. At a point the height of oil is 40m. Find the corresponding height of water at the point.**
   Ans. 36m of water

5. **The diameter of a small piston and a large plunger of a hydraulic jack are 3cm and 10cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :**
   (i) The piston are at the same level
   (ii) Small piston is 40 cm above the large piston

   The density of the liquid in the jack is given as 1000 kg/m^3.

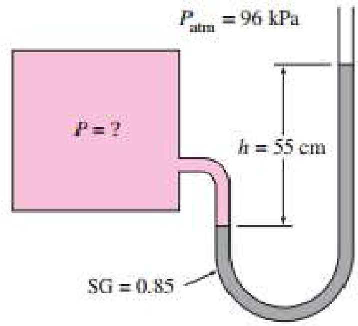
   Ans. (i) 888.96 N, (ii) 919.7 N

6. **The left leg of a U-tube mercury manometer is connected to pipe-line conveying water, the level of mercury in the leg being 0.6m below the centre of pipe-line, and the right leg is open to atmosphere. The level of mercury in the right leg is 0.45m above that in the left leg and the space above mercury in the right leg contains Benzene (sp.gr. 0.88) to a height of 0.3m. Find the pressure in the pipe.**
   Ans.5.784*10^3 kg/m^2.

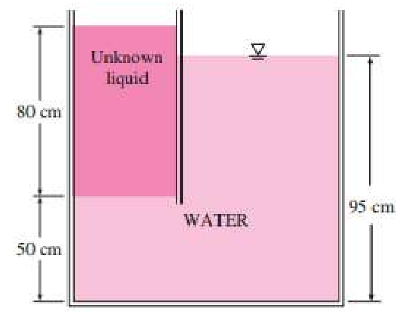
7. A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. Gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm and the height of fluid in the left from the centre of pipe is 15cm below.

Ans.-5.454 N/cm^2.

8. A manometer is used to measure the pressure in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm, as shown in Fig. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.

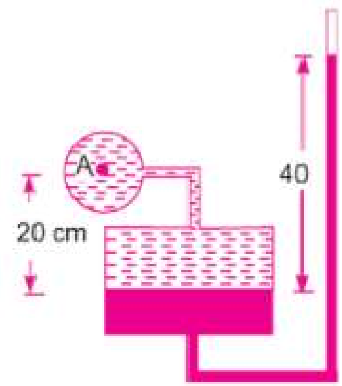


9. The top part of a water tank is divided into two compartments, as shown in Fig. Now a fluid with an unknown density is poured into one side, and the water level raises a certain amount on the other side to compensate for this effect. Based on the final fluid heights shown on the figure, determine the density of the fluid added. Assume the liquid does not mix with water.



SINGLE COLUMN MANOMETER

8. A single column manometer is connected to a pipe containing a liquid of sp.gr. 0.90 as shown in fig. find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in fig. the sp.gr. of mercury is 13.6



Sol: Given

Sp. Gr of the liquid in pipe, $S_1 = 0.9$

∴Density $\rho_1 = 900$ kg/m³.

Sp.gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6*1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20$ cm = 0.20 m

Rise of mercury in right limb, $h_2 = 40$ cm = 0.40m

Let, p_A = pressure in pipe

From equation, we get

$$p_A = \frac{a}{A} h_2 * [\rho_2 *g - \rho_1 *g] + h_2 * \rho_2 *g - h_1 * \rho_1 *g$$

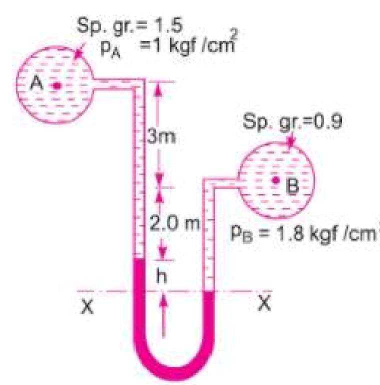
$$= \frac{1}{100} * 0.40 * [13.6*1000*9.81 - 900*9.81] + 0.40*13.6*1000*9.81 - 0.2*900*9.81$$

$$= \frac{0.40}{100}[133416 - 8829] + 53366.4 - 1765.8$$

$= 533.664 + 53366.4 - 1765.8$ N/m². = 52134 N/m². = **5.21 N/cm².**

DIFFERENTIAL MANOMETER

9. A differential manometer is connected at the two points A and B of two pipes as shown in fig. the pipe A contains a liquid of sp.gr. = 1.5 while pipe B contains a liquid of sp.gr. = 0.90. The pressure at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer.



Sol: Given:

Sp.gr. of liquid at A, $S_1 = 1.50$ $\therefore \rho_1 = 1500$ kg/m³.

Sp.gr. of liquid at B, $S_2 = 0.90$ $\therefore \rho_2 = 900$ kg/m³.

Pressure at A, $p_A = 1$ kgf/cm² $= 1 * 10^4$ kgf/m².

$= 10^4 * 9.81$ N/m².

Pressure at B, $p^B = 1.8$ kgf/cm². $= 1.8*10^4*9.81$ N/m².

Density of mercury $= 13.6 * 1000$ kg/m³

Taking X-X as datum line,

Pressure above X-X in the left limb $= 13.6*1000*9.81*h + 1500*9.81*(2+3)+p_A$

$= 13.6*1000*9.81*h + 7500*9.81 + 9.81*10^4$

Pressure above X-X in the right limb = 900*9.81*(h+2)+p_B

= 900*9.81*(h+2)+1.80*10^4*9.81

Equating the two pressures, we get

13.6h + 7.5+ 10 = (h+2.0)*0.9+18

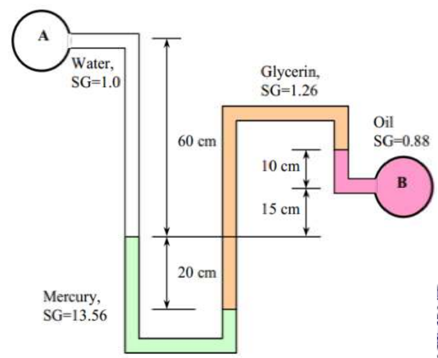
13.6h+17.5 = 0.9*h + 1.8 + 18 = 0.9h + 19.8

(13.6-0.9)h = 19.8 – 17.5 or 12.7h = 2.3

∴ h = 2.3/12.7 = 0.181 m = **18.1 cm**

10. The pressure difference between an oil pipe and water pipe is measured by a double-fluid manometer, as shown in Fig. For the given fluid heights and specific gravities, calculate the pressure difference ΔP = P_B-P_A

Sol:



The pressure difference between two pipes is measured by a double-fluid manometer. For given fluid heights and specific gravities, the pressure difference between the pipes is to be calculated.

Assumption. All the liquids are incompressible.

Properties The specific gravities are given to be 13.5 for mercury, 1.26 for glycerine, and 0.88 for oil. We take the standard density of water to be ρ_w =1000 kg/m^3.

Analysis Starting with the pressure in the water pipe (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the oil pipe (point B), and setting the result equal to P_B give

$P_A + \rho_w g h_w + \rho_{Hg} – \rho_{gly}\, g*h_{gly} + \rho_{oil}\, g*h_{oil} = P_B$

Rearranging and using the definition of specific gravity,

$P_A - P_B = SG_{w*}\rho_{w*}g*h_w + SGHg*\rho_w*g*hHg – SG_{gly} *\rho_w*g*h_{gly} + SG_{oil} *\rho_w *g *h_{oil}$

$= g\, \rho_w (SG_w h_w + SG_{Hg} h_{Hg} – SG_{gliy} h_{gly} + SG_{oil} h_{oil})$

Substituting,

$P_B-P_A = (9.81*1000)*[(1*0.6) + (13.5*0.2) – (1.26*0.45) + (0.88*0.1)] \frac{1\ kN}{1000\ kg.m/s^2}$

P_B-P_A **= 27.7 kN/m²**.

Therefore, the pressure in the oil pipe is 27.7 kPa higher than the pressure in the water pipe.

Discussion Using a manometer between two pipes is not recommended unless the pressures in the two pipes are relatively constant. Otherwise, an over-rise of pressure in one pipe can push the manometer fluid into the other pipe, creating a short circuit.

INVERTED U-TUBE DIFFERENTIAL MANOMETER

11. In Fig., an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp.gr. 0.8. For the manometer readings shown in the fig., find the pressure difference between A and B.

Sol. Given:

Sp. gr. of oil = 0.8 $\therefore \rho_s = 800\ kg/m^3$.

Difference of oil in the oil two limbs = (30+20) – 30 = 20 cm.

Taking datum line at X-X

Pressure in the left limb below X-X = $p_A - 1000*9.81*0.3$

= $p_A - 2943$

Pressure in the right limb below X-X = $p_B - 1000*9.81*0.3 - 800*9.81*0.2$

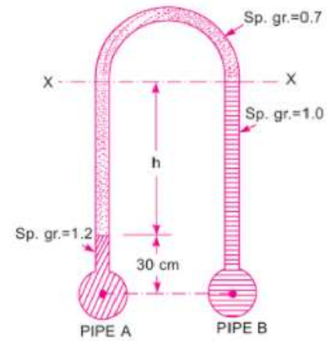
= $p_B - 2943 - 1569.6 = p_B - 4512.6$

Equating the two pressure, $p_A - 2943 = p_B - 4512.6$

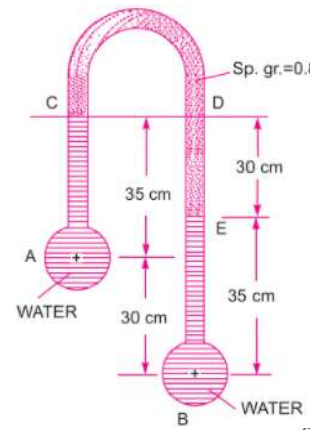
$$p_A - p_B = 4512.6 - 2943 = 1569.6\ N/m^2.$$

Exercise – III (Differential and Inverted Manometers)

1. **Find out the differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as the manometric fluid when connected across pipes A and B as shown in FIG. below, conveying liquids of specific gravity 1.2 and 1.0 and immiscible with manometric fluid. Pipe A and B are located at the same level and assume the pressure at A and B to be equal.**



2. **An inverted U-tube manometer is connected to two horizontal pipes A and B through which water is flowing. The vertical distance between the axes of these pipes is 30cm. when an oil of sp.gr. 0.8 is used as a gauge fluid, the vertical heights of water columns in the two limbs of the inverted manometer (when measured from the respective centre lines of the pipes) are found to be same and equal to 35 cm. determine the difference of pressure between the pipes.**



FLUID STATICS

INTRODUCTION TO FLUID STATICS

Fluid statics deals with problems associated with fluids **at rest**. The fluid can be either gaseous or liquid. Fluid statics is generally referred to as *hydrostatics* when the fluid is a liquid and as *aerostatics* when the fluid is a gas. In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses ($\tau = 0$)in the fluid trying todeform it. The only stress we deal with in fluid statics is the *normal* stress, which is the pressure, and the variation of pressure is due only to the weight of the fluid. Therefore, the topic of fluid statics has significance only in gravity fields, and the force relations developed naturally involve the gravitational acceleration g.The force exerted on a surface by a fluid at rest is normal to the surface at the point of contact since there is no relative motion between the fluid and the solid surface, and thus no shear forces can act parallel to the surface.

Fluid statics is used to determine the forces acting on floating or submerged bodies and the forces developed by devices like hydraulic presses and car jacks. The design of many engineering systems such as water damsand liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics. The complete description of the resultant hydrostatic force acting on a submerged surface requires the determination of the magnitude, the direction, and the line of action of the force.

Definitions

Total Pressure and Centre of Pressure

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface.

There are four cases of

HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be:

1. Horizontal plane surface,
2. Vertical plane surface,
3. Inclined plane surface and
4. Curved surface.

Horizontal plane surface submerged in liquid

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to,

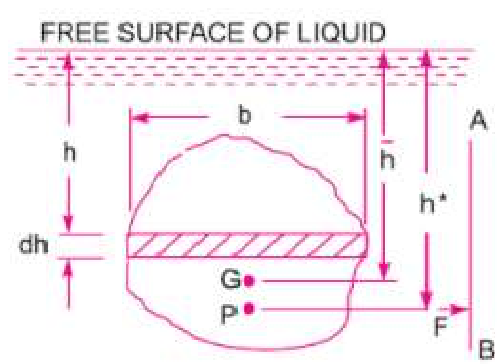
p = $\rho g\bar{h}$**,** where h is depth of surface

Let, A = total area of surface

Then total force, F, on the surface = p*Area = $\rho g * \bar{h} * A$

Where $\bar{h}$ = depth of C.G. from free surface of liquid = h

h* = depth of centre of pressure from free surface = h



Total pressure and Centre of pressure for vertically submerged surface:

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in FIG

Let, A = total area of the surface

$\bar{h}$ = distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h* = Distance of centre of pressure from free surface of liquid

(a) Total Pressure (F): The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strips is then calculated and the total pressure force on the whole area is calculated by integrating the forces on small strip.

Considering a strip of thickness 'dh' and width 'b' at a depth of 'h' from free surface of liquid as shown in fig.

Pressure intensity on the strip, $p = \rho * g * h$

Area of the strip, $dA = b * dh$

Total pressure force on the whole surface,

$$F = \int dF = \int \rho * g * h * b * dh = \rho g \int b * h * dh$$

But $\int b * h * dh = \int h * dA$

= Moment of surface area about the free surface of liquid

= Area of surface * Distance of C.G. from free surface

$= A * \bar{h}$

∴ $F = \rho * A * \bar{h}$

(b) Centre of Pressure (h*):

Centre of pressure is calculated by using the "Principle of moments", which states that the moment of the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance of h* from the free surface of the liquid as shown in fig. hence moment of the force F about free surface of the liquid = F x h*

Moment of force dF, acting on a strip about free surface of liquid

$= dF * h$

$= \rho g h * b * dh * h$

Sum of moments of all such forces about free surface of liquid

$= \int \rho g h * b * dh * h = \rho g \int b * h * h dh$

$= \rho g \int b * h^2 * dh = \rho g \int h^2 * dA$

But, $= \int h^2 * dA = \int b h^2 * dh$

= Moment of inertia of the surface about free surface of liquid

$= I_o$

∴ Sum of moments about free surface $= \rho g I_o$

Equating the equations, we get

$$F \times h^* = \rho g I_o$$

But, $F = \rho g A \bar{h}$

$\therefore \rho g A\bar{h} * h^* = \rho g I_o$

$$h^* = \frac{\rho g\, I_o}{\rho g\, A\bar{h}} = \frac{I_o}{A\bar{h}}$$

By the theorem of parallel axis, we have

$$I_o = I_G + A * \overline{h^2}$$

Where I_G = moment of inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_o in equation, we get

$$h^* = \frac{I_G + A\overline{h^2}}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

In the above equation, $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation, it is clear that:

(I) Centre of pressure lies below the centre of gravity of the vertical surface.

(II) The distance of centreof pressure from the free surface of liquid is independent of the density of the liquid.

Table:Centre of Gravity and Moment of Inertia for some typical shapes

Shape	CG	I_G	I_{base}
1. Triangle, side b height h and base zero of x axis	$h/3$	$bh^3/36$	$bh^3/12$
2. Triangle, side b height h and vertex zero of x axis	$2h/3$	$bh^3/36$	$bh^3/12$
3. Rectangle of width b and depth D	$D/2$	$bD^3/12$	$bD^3/3$
4. Circle	$D/2$	$\pi D^4/64$	–
5. Semicircle with diameter horizontal and zero of x axis	$2D/3\pi$	–	$\pi D^4/128$
6. Quadrant of a circle, one radius horizontal	$4R/3\pi$	–	$\pi R^4/16$
7. Ellipse : area $\pi bh/4$ Major axis is b, horizontal and minor axis is h	$h/2$	$\pi bh^3/64$	–
8. Semi ellipse with major axis as horizontal and $x = 0$	$2h/3\pi$	–	$\pi bh^3/128$
9. Parabola (half) area $2bh/3$ (from vertex as zero)	$y_g = 3h/5$ $x_g = 3b/8$	–	$2bh^3/7$

Total pressure and Centre of pressure on Inclined plane surface submerged in liquid:

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in FIG.

Let, A = total area of the inclined surface,

$\bar{h}$ = distance of C.G. of the inclined area from free surface of liquid

h* = Distance of centre of pressure from free surface of liquid

θ = Angle made by the plane of the surface with free liquid surface

Total pressure

Let the plane of the surface, if produced meet the free liquid surface and at a distance y from the axis O-O as shown in FIG

Pressure intensity on the strip p = ρgh

∴pressure force, dF, on the strip, dF = p*area of strip = ρgh*dA

Total pressure force on the whole area, F = $\int dF = \int \rho gh * dA$

But from FIG. $\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$ = sinθ

∴ h = y sinθ

∴ F = $\int \rho * g * y * \sin\theta$ *dA = $\rho g \sin\theta \int y * dA$

But $\int y * dA$ = A * $\bar{y}$

Where $\bar{y}$ = Distance of C.G. from axis O-O

∴ F = ρg*sinθ * $\bar{y}$ *A

$F = \rho g A\bar{h}$

Centre of pressure (h*)

Pressure force on the strip, $dF = \rho gh\, dA = \rho gy\sin\theta dA$

Moment of the force, dF, about axis O-O = $dF * y = \rho gy\sin\theta dA * y = \rho gy^2\sin\theta dA$

Sum of moments of all such forces about O-O

$$= \int \rho g y^2 \sin\theta\, dA = \rho g \sin\theta \int y^2 dA$$

But $\int y^2 dA$= M.O.I of the surface about O-O = I_o

∴ Sum of moments of all forces about O-O = $\rho g \sin\theta I_o$

Moment of the total force, F, about O-O is also given by

$$= F \times y^*$$

Where y* = distance of centre of pressure from O-O

Equating the two values given by equations

$$F \times y^* = \rho g \sin\theta I_o$$

$$y^* = \frac{\rho g \sin\theta\, I_o}{F}$$

Now $y^* = \frac{h^*}{\sin\theta}$, $F = \rho g A\bar{h}$

And I_o by the theorem of parallel axis = $I_G + A\,\overline{y^2}$.

Substituting these values in equation, we get

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta\, I_o}{\rho g A\bar{h}} [I_G + A\,\overline{y^2}.]$$

But $\frac{\bar{h}}{\bar{y}} = \sin\theta \Rightarrow \bar{y} = \frac{\bar{h}}{\sin\theta}$

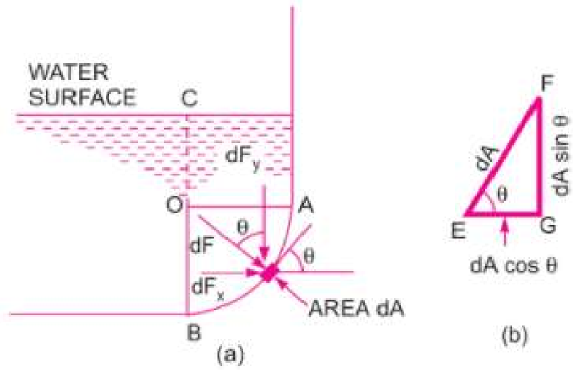
∴ $$h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A \times \frac{\overline{h^2}}{\sin^2\theta}\right]$$

∴ $$\mathbf{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

In above equation, I_G = M.O.I. of inclined surfaces about an axis passing through G and parallel to O-O.

Total pressure and Centre of pressure on Curved surface submerged in liquid:

Consider a curved surface AB, submerged in a static fluid as shown in FIG. Let dA is the area of a small strip at a depth of h from water surface.



Then pressure intensity on the area dA is = ρgh

and pressure forces, dF = p x Area = ρgh * dA

This force dF acts normal to the surface

Hence total pressure force on the curved surface should be

$F = \int \rho gh \, dA$

But here as the direction of the forces on the small area are not in the same direction, but varies from point to point. Hence integration of equation for curved surface is impossible. The problem can, however, be solved by resolving the force dF in two components $dF_x$and dF_y in the x and y directions respectively. The total force in the x and y directions, i.e., F_x and F_y are obtained by integratingdF_x and dF_y. Then total force on the curved surface is

$$F = \sqrt{F_x^2 + F_y^2}$$

and inclination of resultant with horizontal is $\tan \phi = \frac{F_y}{F_x}$

Resolving the forces dF given by equation in x and y directions:

$$dF_x = dF\sin\theta = \rho gh \, dA \sin\theta$$

$$dF_y = dF\cos\theta = \rho gh \, dA \cos\theta$$

Total forces in the x and y directions are:

$$F_x = \int dF_x = \int \rho gh \, dA \sin\theta = \rho g \int h dA \sin\theta$$

$$F_y = \int dF_y = \int \rho gh \, dA \cos\theta = \rho g \int h dA \cos\theta$$

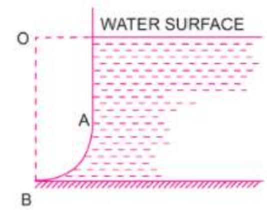
FIG. shows the enalarged area dA. From this figure, i.e., ΔEFG

EF = dA

FG = dAsinθ

EG = dAcos θ

Thus in equation, dAsinθ = FG = Vertical projection of the area dA and hence the expression $\rho g \int hdA \sin\theta$ represents the total pressure force on the projected area of the curved surface on the vertical plane. Thus



F_x = total pressure force on the projected area of the curved surface on vertical plane

Also dAcosθ= EG = horizontal projection of dA and hence had cos θ is the volume of the liquid contained in the elementary area dAup to free surface of the liquid. Thus $\int hdA \cos\theta$ is the total volume contained between the curved surface extended upto free surfaces.

Hence $\rho g \int hdA \cos\theta$ is the total weight supported by the curved surface. Thus

$$F_y = \rho g \int hdA \cos\theta$$

= weight of liquid supported by the curved surface upto free surface of liquid.

In FIG., the curved surface AB is not supporting any fluid. In such cases, F_y is equal to the weight of the imaginary liquid supported by AB upto free surface of liquid. The direction of F_y will be taken in upward direction.

PROBLEMS

1. **A square surface 3 m * 3 m lies in a vertical plane. Determine the position of the centre of pressure and the total force on the surface, when its upper edge is (a)in water surface and (b) 15m below the water surface.**

Sol: (a) Total force on the square

$F = g\bar{h}A = 1000*9.81*1.5*9 = 132.3$ kN

The location of total force is given by

$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where $\bar{h}$ **and** h^* are the distance of C.P. and centroid below the water surface.

$= \frac{\frac{1}{12}*3*3^3}{9*1.5} + 1.5 = 2.0$ m

(b) Total force on the square,

$F = g\bar{h}A = 1000*9.81*(1.5*15)*9 = 1457$ kN

Distance between centre of pressure and centroid $= \bar{h} - h^* = \frac{I_G}{A\bar{h}}$

$= \frac{\frac{1}{12}*3*3^3}{9*16.5}$

$= 0.0455$ m $= 4.55$ cm

Depth of centre of pressure = 16.5 + 0.0455 = 16.5455 m.

Exercise - I

Q. a rectangular plane surface is 2m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5m below the free surface.

Ans. (a) F = 88290 N; h* = 2.0 m, (b) F = 235440 N; h* = 4.1875 m

Q. Determine the total pressure ona circular plate of diameter 1.5m which is placed vertically in water in such a way that the centre of the plate is 3m below the free surface of water. Find the position of centre of pressure also.

Ans. F = 52.002 kN, h* = 3.046 m

Q. Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4m and altitude 4m when it is immersed vertically in an oil of sp.gr. 0.9. The base of the palte coincides with the free surface of oil.

Ans. F = 9.597 kN, h* = 1.99m

Q. A vertical sluice gate is used to cover an opening in a dam. The opening is 2 m wide and 1.2 m high. On the upstream of the gate, the liquid of sp. gr. 1.95, lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it Assume that the gate is hinged at the bottom.

Ans. Resultant = 57.565 kN at 0.578 m above the hinge.

Q. A caisson for closing the entrance to a dry dock is of trapezoidal form 16 m wide at the top and 10 to wide at the bottom and 6 m deep. Find the total pressure and centre of pressure on the caisson if the water on the outside is just level with the top and dock is empty.

Ans. F = 2.118 MN, h* = 3.833 m

2. **A rectangular plane surface 2 in wide and 3 in deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of centre of pressure when the upper edge is 1.5 m below the free water surface.**

Sol. Given:

Width of plane surface, b = 2 m

Depth, d = 3 m

Angle, θ = 30°.

Distance of upper edge from free water surface = 1.5m

(i) Total pressure force is given by equation as

$$F = \rho g A\bar{h}$$

where ρ = 1000 kg/m^3.

A = b *d = 3 * 2 = 6m

$\bar{h}$ = Depth of C.G. from free water surface = AE + EB = 1.5+ BC sin 30°

= 1.5 + 1.5 sin 30°. = 2.25 m

∴ **F = 132.435 kN**

(ii) Centre of pressure (h*)

Using the equation, we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}, \text{ where } I_G = bd^3/12 = (2*3^3)/12 = 4.5\ m^4.$$

$$\therefore \quad h^* = \frac{4.5 \sin^2 30^o}{6*2.25} + 2.25 = \mathbf{2.33\ m}$$

Q. A rectangular plane surface 3 m wide and 4 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure force and position of centre of pressure, when the upper edge is 2 m below the free surface.

Ans. T.P. = 353.167 kN, C.P position = 3.111 m

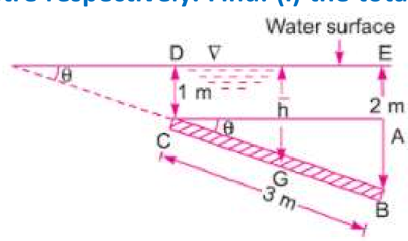
Q. A circular plate 3.0 m diameter is immersed in water in such a way that its greatest and least depth below the free surface are 4 m and 1.5 m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.

Ans. T.P. = 190.62 kN, C.P. position =2.891 m

Q. If in the above problem, the given circular plate is having a concentric circular hole of diameter 1.5 m, then calculate the total pressure and position of the centre of pressure on one face of the plate.

Ans. T.P. = 143.018 kN, C.P.position = 2.927

Q. A circular plate 3 metre diameter is submerged in water as shown in Fig. Its greatest and least depths are below the surfaces being 2 metre and 1 metre respectively. Find: (i) the total pressure on front face of the plate, and (ii) the position of centre of pressure.



Ans. F = 104.013 kN, h* = 1.54 m

Q. Find the total pressure and position of centre of pressure on a triangular plate of base 2 m and height 3 m which is immersed in water in such a way that the plane of the plate makes an angle of 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5 m from water surface.

Ans. F = 99.0661 kN, h* = 3.477 m

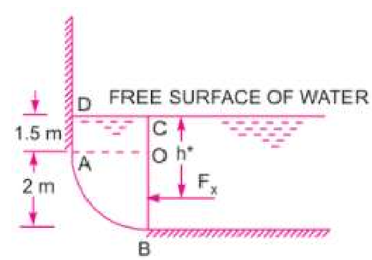
3. **Compute the horizontal and vertical components of the total force acting on a curved surface AB, which is in the form of a quadrant of a circle of radius 2 m as shown in Fig. Take the width of the gate as unity.**

Sol: Given:

Width of gate = 1.0 m

Radius of the gate = 2.0 m

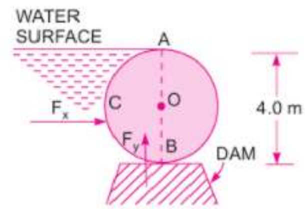
∴ Distance AO = OB = 2 m



Horizontal force, F_x exerted by water on gate is given by surface AB on vertical plane = Total pressure force on OB

{Projected are of curved surface on vertical plane = OB * 1}

$= \rho g A\bar{h}$

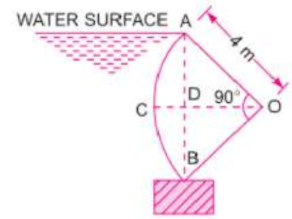


$= 1000*9.81*2*1* \left(1.5 + \frac{2}{2}\right)$

{∵ Area of OB = A = BO*1 = 2*1 = 2, $\bar{h}$ = Depth of C.G. of OB from free surface = 1.5 + (2/2)}

F_x = 9.81 * 2000*2.5 = **49.05 kN**

The point of application of F_x is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$



Where, IG = M.O.I of OB about its C.G. = $\frac{bd^3}{12} = \frac{1*2^3}{12} = \frac{2}{3}$ m⁴.

∴ $h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{2}{3}}{2*2.5} + 2.25$ = **2.633 from free surface**

Vertical force, F_y exerted by water is given by equation

F_y = Weight of water supported by AB upto free surface

= Weight of portion DABOC

= Weight of DAOC + Weight of water of AOB

= ρg [Volume of DAOC + Volume of AOB]

= 1000*9.81 [AD*AO*1 + $\frac{\pi}{4}(AO)^2 * 1$]

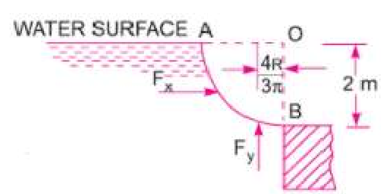
= 1000*9.81 [1.5*2.0*1 + $\frac{\pi}{4}(2)^2 * 1$] = **60.249 kN**

Q. Fig.shows a gate having a quadrant shape of radius 2m. Find the resultant force due to water per metre length of the gate. Find also the angle at which the total force will act.

Ans. Resultant force = 36.534 kN, θ = 57° 31'.

Q. Find the magnitude and direction of the resultant force due to water acting on a roller gate of cylindrical form of 4.0 m diameter, when the gate is placed on the dam in such a way that water is just going to spill. Take the length of the gate as 8 m.

Q. Find the horizontal and vertical component of water pressure acting on the face of a tainter gate of 90o sector of radius 4m as shown in FIG. take gate width as unity



4. **A cylindrical gate of 4m diameter 2m long has water on its both sides as shown in figure. Determine the magnitude, location and direction of the resultant force exerted by the water on the gate. Find also the least weight of the cylinders so that it may not be lifted away from the floor.**

Sol. Given:

Dia of gate = 4m

Radius = 2 m

(i) The forces acting on the left side of the cylinder are:
The horizontal component, F_{x1}
Where F_{x1} = Force of water on area projected on vertical plane
= Force on area AOC
= $\rho g A\bar{h}$ [∵A = AC*width = 4 *2 = 8, $\bar{h}$ = (1/2)*4 = 2 m)]
= 1000*9.81*8*2
= **156960 N**

F_{y1} = weight of water enclosed by ABCOA
= 1000 * 9.81 *$\frac{\pi}{4}$ * R^2* 2 = 1000 * 9.81 *$\frac{\pi}{4}$ * 2^2 * 2 = **123276 N**.

Right side of the cylinder

Fx2 = $\rho g A2\bar{h}_2$ = Force on vertical area CO

= 1000 * 9.81 * 2 * (2/2)

∵ {A_2 = CO * 1 = 2 * 1 = $2m^2$, $\bar{h}_2$ = (2/2) = 1.0}

= 39.240kN

Fy2 = Weight of water enclosed by DOCD

= ρg* $\frac{\pi}{4}$ R^2 * width of gate

= 1000*9.81* $\frac{\pi}{4}$ 2^2 * 2 = 61.638 kN

∴ Resultant force in the direction of x,

Fx = Fx1 – Fx2 = 156.960 – 39.240 = 117.720 kN

Resultant force in the direction of y,

Fy = Fy1 + Fy2 = 123.276 + 61.638 = 184.914 kN

(i) Resultant force, F is given as

F = $\sqrt{F_x^2 + F_y^2}$ = $\sqrt{117.720^2 + 184.914^2}$ = **219.206 kN**

(ii) Direction of resultant force is given by

tan θ = $\frac{F_y}{F_x}$ = $\frac{184.914}{117.720}$ = 1.5707

θ = 57^o 31'.

(iii) Location of the resultant force

Force, Fx1 acts at a distance of (2*4)/3 = 2.67 m from the top surface of water on left side, while Fx2 acts at a distance of (2*3)*2 = 1.33 m from free surface on the right side of the cylinder. The resultant force Fx in the direction of x will act at a distance of y from the bottom as

Fx * y = Fx1 [4 – 2.67] – Fx2 [2-1.33]

117.720 * y = 156.960*1.33 – 392.240-0.67 = 208.756.8 – 26.290 = 182.466

∴ $y = \frac{182.466}{117.720}$ = 1.55 m from the bottom

Force Fy1, acts at a distance $\frac{4R}{3\pi}$ = 0.8488 m from AOC towards the right of AOC. The resultant force Fy will act at a diatnce x from AOC which is given by

Fy * x = Fy1 * 0.8488

184.914 * x = 123.276 * 0.8488 – 61.638 * 0.8488 = 0.8488 [123.276 – 61.638]

= 52.318 kN

∴ x = 0.2829 m from AOC

(iv) Least weight of cylinder. The resultant force in the upward direction is

Fy = 184.914 kN

Thus the weight of cylinder should not be less than the upward force Fy. Hence least weight of the cylinder should be at least **= 184.914 kN**

BUOYANCY, FLOTATION AND METACENTRE

Archimedes'Principle

When a stationary body is completely submerged in a fluid or floating so that it is only partially submerged, the resultant fluid force acting on the body is called the **buoyant force.**

Note that the forces F1, F2, F3, and F4 are simply the **forces** exerted on the plane surfaces, **W(=mg)** is the weight of the shaded fluid volume, and F_B is the force the body is exerting on the fluid.

The forces on the vertical surfaces, such as F3 and F4, are all equal and cancel, so the equilibrium equation of interest is in the z-direction and can be expressed as

$F_B = F_2 - F_1 - mg$

If the specific weight of the fluid is constant, then ;

$$F_2 - F_1 = \rho g\,(h_2 - h_1) \qquad (1)$$

where **A** is the horizontal area of the upper (or lower) surface, and Equation (1) can be written as :

$$F_B = \rho g\,(h_2 - h_1)\,A - \rho g\,[(h_2 - h_1)\,A - V]$$

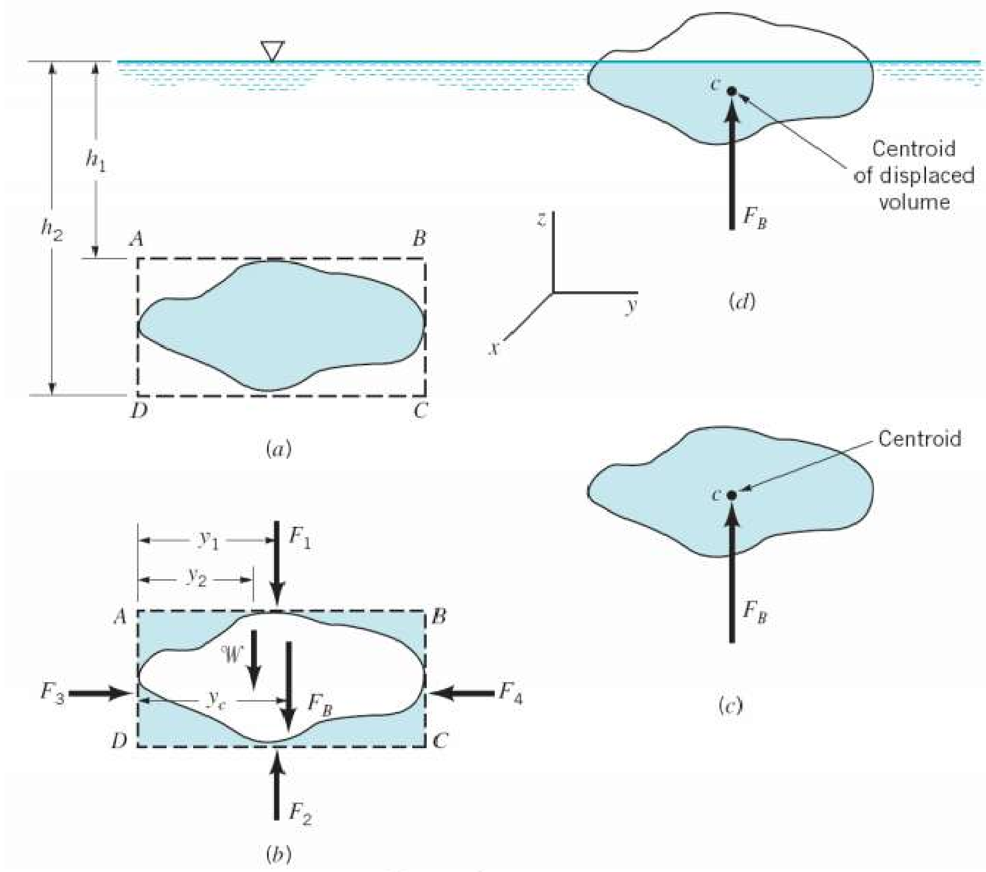
Simplifying, we arrive at the desired expression for the buoyant force:

$$F_B = \rho g\,V \quad \text{[i.e., Weight of fluid displaced by the body (Upward)]}$$

Archimedes' principle states that *"the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward.''*

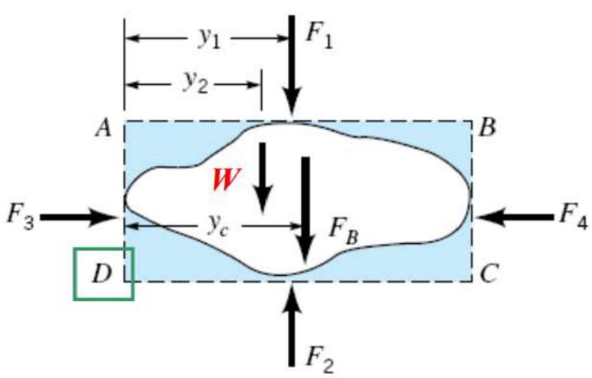
Thus, we conclude that the buoyant force passes through the centroid of the displaced volume as shown in Figure 1(c).

The point through which the buoyant force acts is called the **center of buoyancy.**



location of F_B

Consider a moment equation about an axis passing through D (x-axis)



Moment = $F_2y_1 – F_1y_1 –Wy_2-F_By_c = 0$ (where y_c is to be determined)

$\Upsilon h_2 A y_1 - \Upsilon h_1 A y_1 - \Upsilon [(h_2-h_1)A – V]y_2 - \Upsilon V y_c = 0$

$(h_2 - h_1)A y_1 - [(h_2-h_1)A – V]y_2 - V y_c = 0$

(where h_2-h_1 = total volume of parallelepiped, V_T)

$V_T\ y_1 = V^*y_c + (V_T - V)\ y_2$ (equation of volume center)

Where y_1 = centre of total volume,

y_c = center of displace volume,

y_2 = center of fluid volume in parallelepiped

∴ y_c : y coordinate of the centroid of displaced volume V

By a similar manner,

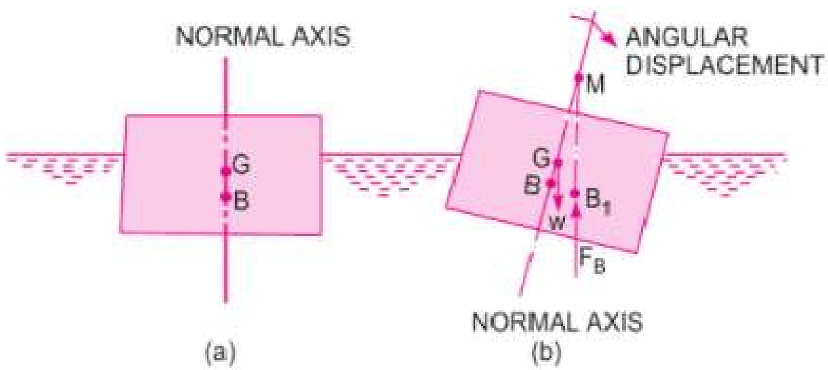
∴ x_c: x coordinate of the centroid of displaced volume V

METACENTRE

It is defined as ***the point about which a body starts oscillating when the body is tilted by a small angle***.

The meta-centre may also be defined as ***the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.***

Consider a body floating in a liquid as shown in Fig. (a). Let the body is in equilibrium and **G** is the centre of gravity and **B** the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.



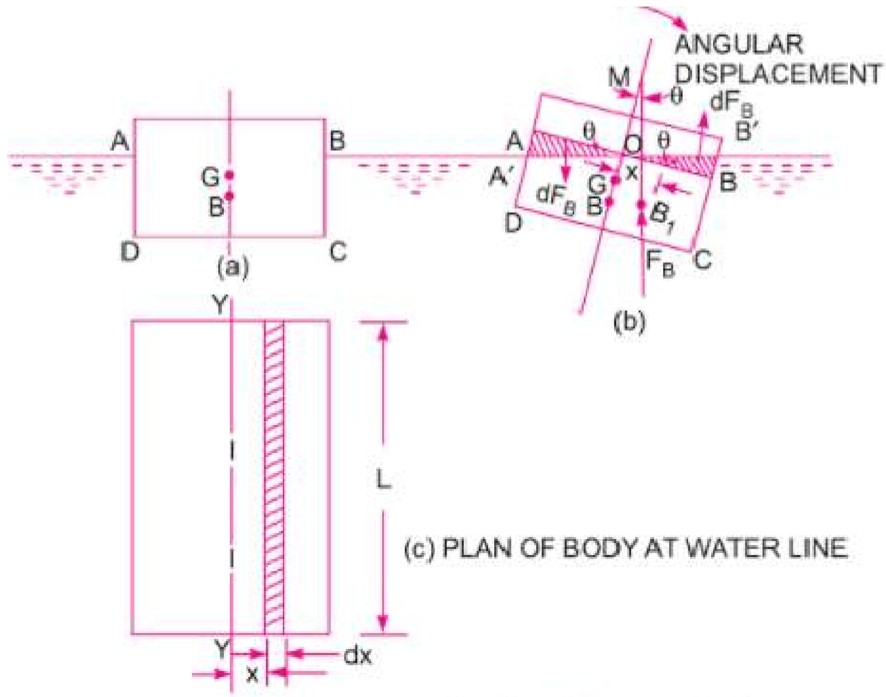
Let the body is given a small angular displacement in the clockwise direction as shown in Fig. (b). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig.(b). The line of action of the force of buoyancy in this new position will intersect the normal axis of the body at some point say **M**. This point **M** is called **Meta-centre.**

Metacentric Height

The distance MG, ie., ***the distance between the met-centre of a floating body and the centre of gravity of the body*** is called **meta-centric** height.

Analytical Method For Metacentric Height

Fig.(a) shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at **G** and **B**. The floating body is given a small angular displacement in the clockwise direction. This is shown in Fig.(b). The new centre of buoyancy is at $\mathbf{B_1}$. The vertical line through $\mathbf{B_1}$ cuts the normal axis at **M**. Hence **M** is the meta-centre and **GM** is meta-centric height.



The angular displacement of the body in the clockwise direction causes the wedge-shaped prism BOB' on the right of the axis to go inside the water while the identical wedge-shaped prism represented by AOA' emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the C.G. of the prism BOB' while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of AOA'. The couple due to these buoyant forces dF_B tends to rotate the ship in the counterclockwise direction. Also the moment caused by the displacement of the centre of buoyancy from B to B_1 is also in the counterclockwise direction. Thus these two couples must be equal.

Couple due to wedge:

Consider towards the right of the axis a small strip of thickness dx at a distance x from O as shown in Fig.(b).

The height of strip $x * \angle BOB' = x * \theta$

$$\{\because \angle BOB' = \angle AOA' = BMB_1' = \theta\}$$

$\therefore$ Area of strip = height * thickness = $x * \theta * dx$

If L is the length of the floating body, then

Volume of strip = area * L

= x * θ * L * dx

∴ Weight of strip = ρg * volume = ρg*x*θ*L*d_x

Similarly, if a small strip of thickness d_x at a distance x from O towards the left of the axis is considered, the weight of the strip will be ρg*x*θ*L*d_x. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = weight of each strip * distance between these two weights

= ρg*x*θ*L*d_x[x +x]

= ρg*x*θ*L*d_x*2x = 2 ρg*x^2*θ*L*d_x

∴ moment of the couple for the whole wedge

$$= \int 2\,\rho g * x^2 * \theta * L * dx \qquad (1)$$

Moment of couple due to shifting of centre of buoyancy from B to B1

= F_B * BB_1

= F_B * BM * θ {∵ BB_1 = BM * θ if θ is very small}

= W * BM * θ {∵ F_B = W} (2)

But these two couples are the same. Hence equating the equations (1) and (2), we get

$$W * BM * \theta = \int 2\,\rho g * x^2 * \theta * L * dx$$

$$W * BM * \theta = 2\,\rho g\theta \int x^2 * L * dx$$

$$W * BM = 2\,\rho g \int x^2 * L * dx$$

Now Ld_x = Elemental area on the water line shown in fig (c) and = dA

$$\therefore \quad W * BM * \theta = 2\,\rho g * \int x^2 * dA$$

But from Fig(c) it is clear that $2 \int x^2 * dA$ is the second moment of area of the plan of the body at water surface about the axis Y-Y. Therefore

$$W * BM = \rho g I \qquad \{\text{where } I = 2\int x^2 * dA\}$$

$$\therefore \quad BM = \frac{\rho g I}{W}$$

But W = weight of the body

= Weight of the fluid displaced by the body

= ρg* Volume of the fluid displaced by the body

$= \rho g *$ Volume of the body submerged in water

$= \rho g * \forall$

$\therefore \quad BM = \frac{\rho g * I}{\rho g * \forall} = \frac{I}{\forall}$ **(3)**

$GM = BM - BG = \frac{I}{\forall} - BG$

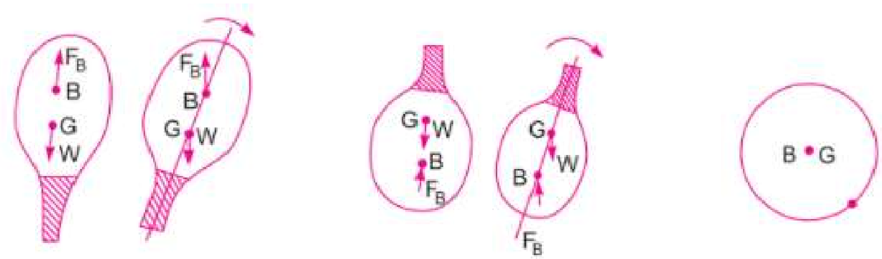
$\therefore$ **Meta- centric height = GM =** $\frac{I}{\forall} - BG$ **(4)**

CONDITIONS OF EQUILIBRIUM OF A FLOATING AND SUB-MERGED BODIES

A sub-merged or a floating body is said to be stable if it comes back to its original position after a slight disturbance. The relative position of the centre of gravity (G) and centre of buoyancy (B1) of a body determines the stability of a sub-merged body.

Stability of a Sub-merged Body.

The position of centre of gravity and centre of buoyancy in case of a completely sub-merged body are fixed. Consider a balloon, which is completely sub-merged in air. Let the lower portion of the balloon contains heavier material, so that its centre of gravity is lower than its centre of buoyancy as shown in Fig.(a). Let the weight of the balloon is W. The weight W is acting through G, vertically in the downward direction, while the buoyant force F_B is acting vertically up, through B. For the equilibrium of the balloon W= F_B. If the balloon is given an angular displacement in the clockwise direction as shown in Fig. (a), then W and FR constitute a couple acting in the anti-clockwise direction and brings the balloon in the original position. Thus the balloon in the position, shown by Fig. (a) is in stable equilibrium.



(a) STABLE EQUILIBRIUM (b) UNSTABLE EQUILIBRIUM (c) NEUTRAL EQUILIBRIUM

(a) Stable Equilibrium.

When W = FB and point B is above G, the body is said to be in stable equilibrium.

(b) Unstable Equilibrium.

If W = F s, but the centre of buoyancy (B) is below centre of gravity (G), the body is in unstable equilibrium as shown in Fig. (b). A slight displacement to the body, in the clockwise direction, gives the couple due to W and F_B also in the clockwise direction.

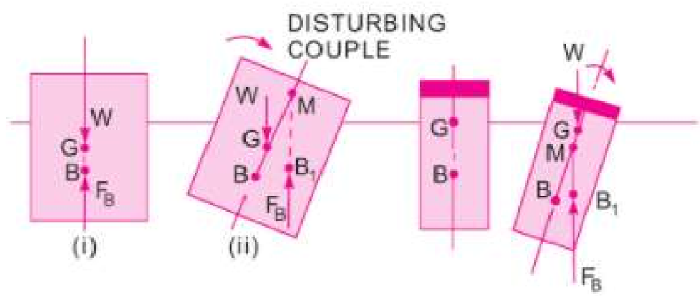
Thus the body does not return to its original position and hence the body is in unstable equilibrium.

(c) Neutral Equilibrium.

If F_B = W and B and G are at the same point, as shown in Fig.(c), the body is said to be in neutral equilibrium.

Stability of Floating Body.

The stability of a floating body is determined from the position of Meta-centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.



(a) Stable equilibrium M is above G (b) Unstable equilibrium M is below G.

(a) Stable Equilibrium.

If the point M is above G, the floating body will be in stable equilibrium as shown in Fig. (a). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through Bi cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.

(b) Unstable Equilibrium.

If the point M is below G, the floating body will be in unstable equilib-rium as shown in Fig. (b). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(c) Neutral Equilibrium.

If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.