UNIT-2

COLUMNS

A member is subjected to compression is termed as post, stanchion, strut or column/pillar. though there is no rigid distinction in the usage of terms. Wooden columns as posts; built-up rolled steel sections as stanchions & compression members are called as struts. (inclined members)

The main supporting members of building are generally called as columns or pillars irrespective of the materials that are made of. Column is a compression member.

Load carried by a short column is given by the relation

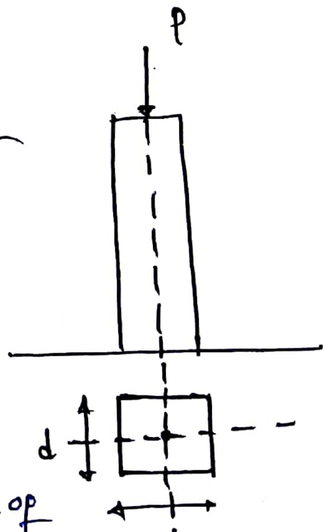
$$P = \sigma \times A$$

where 'σ' is the intensity of stress ; A c/s area of the column.

When 'σ' be the ultimate crushing strength of the column material

$$\sigma = \sigma_{max} = \sigma_u$$

then 'P' shall give the crushing load for the column.



But then columns don't generally fail only due to crushing. They buckle too.

Let us consider a wooden section 6cm square (6cm×6cm) & 40cm long, carrying some axial load. The max. load carried by another wooden member of equal cross-section, say 9cm×4cm & of the same ht. of 40cm. shall obviously be lesser than that in the first case. Further keeping the area of c/s the same but changing the size to 18cm × 2cm. the load-carrying capacity shall be heavily reduced.

Column: A structural member subjected to axial compressive force is called as column. Normally, columns carry heavy compressive loads. Columns used in concrete & steel buildings.

Types of columns: Columns are classified based on slenderness ratio (λ)

$$\text{slenderness ratio}(\lambda) = \frac{L_e}{r_{min}} = \sqrt{\frac{I}{A}}$$

$$\text{slenderness ratio} = \frac{\text{Ht of the column}}{\text{Least lateral dimension of the column}}$$

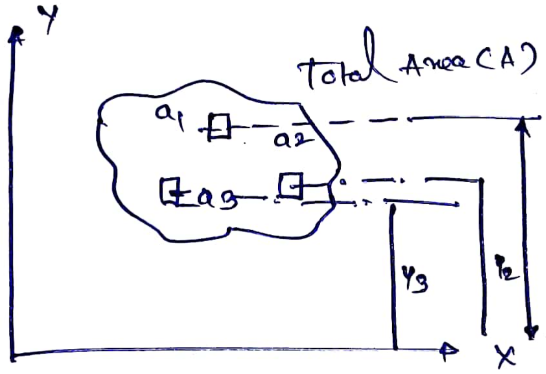
$$\lambda = \frac{\text{Ht of the column (Effective length of the column)}}{\text{Least radius of gyration}}$$

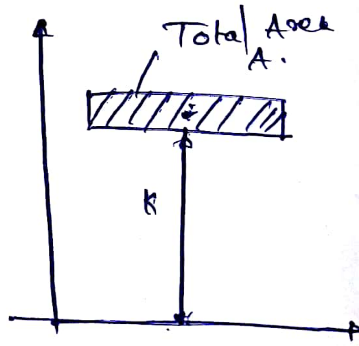
Radius of gyration: distance from a given axis upto a point where the entire area is assumed to be concentrated

First moment of area = AK

second moment of area = $\boxed{I = AK^2}$

$$K = \sqrt{\frac{I}{A}}$$





Moment of area about x-axis

First $I_{xx} = AK$

Again taking the moment of area about x axis

$$I_{xx} = AK^2$$

$$\Rightarrow K = \sqrt{\frac{I_{xx}}{A}}$$

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Short columns: When the length of column is less as compared to its c/s dimensions it is called short column.

$$\lambda = \frac{L_e}{r_{min}} < 50$$

Crushing load: The load at which short column fails by crushing is called crushing load.

$\frac{I}{A} = r^2$

Intermediate columns: It ranges from 50 – 200.

Critical slenderness ratio $30 < \frac{L_e}{r_{min}} < 100$

Long columns: λ exceeds more than 200

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In this condition, the column is unstable & is regarded as having failed.

$$\text{Safe load / working load} = \frac{\text{Crippling load}}{F.O.S}$$

A column that which fails primarly due to direct stress σ_d is called short column.

A column that which fails primarly due to bending stress σ_b is called the long column.

short columns: length of column < 8 times of diameter (smallest side)

$$\lambda < \frac{\text{length of col}}{\text{smallest side}}$$

$$\lambda < 32$$

Medium columns: Columns with slenderness ratio more than 32 but lesser than 120 or with length more than 30 times the diameter.

$$32 < \lambda < 120 \quad \text{medium columns}$$

Long Columns: Columns with slenderness ratio is more than 120 or, whose length is more than 30 times the diameter

The columns may further buckle because of one or more of the following reasons.

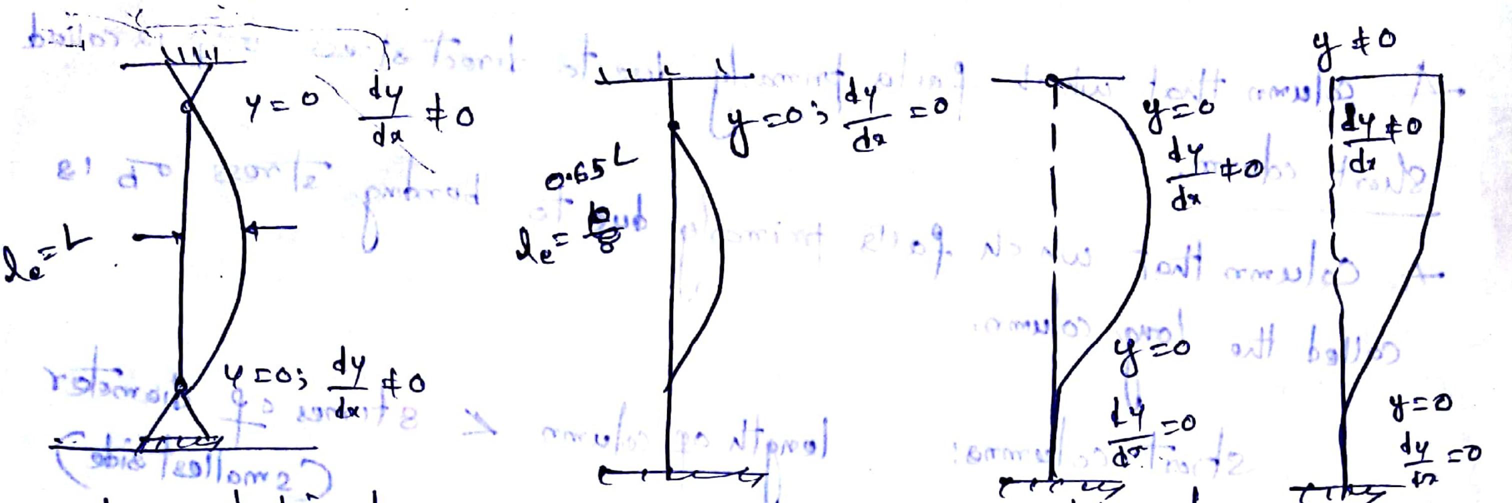
(i) It may not initially ~~be~~ perfectly straight

(ii) The load may not be applied along the axis of the column.

(iii) The materials properties (or) behaviour may not be homogeneous.

End conditions:

The manner of securing the ends are termed as end conditions / end connections here, 'y' is the lateral displacement of vertical axis.



both ends hinged | Both ends fixed | one end hinged other end fixed | one end fixed one end free

24/1/20

141, 146, 149, 154

166, 174, 175

178, 179, 132, 144

140, 147, 148, 149

155, 156, 159

Apart from slenderness, the load carrying capacity of column depends upon its end conditions too.

Buckling produced in a column by a load smaller than the crippling load, & removal of the load then column straightens ~~stra~~ back to its original unloaded condition indicating a state of static equilibrium. However, if the load increases gradually, then the stage is reached when on removal of load, the column doesn't wholly straighten to its unloaded position.

The load at this stage when the column just retains a permanent set & doesn't fully return back to its original unloaded state is called critical crippling or buckling load.

Linear differential equation of nth order (Higher order)

The nth order differential eqn

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + P_n y = X$$

where $P_1, P_2, \ldots P_n$ are functions of x only

The operator $\frac{d}{dx}$ is denoted by D. ✓

$$\therefore \quad D^n y + P_1 D^{n-1} y + P_2 D^{n-2} y + \cdots + P_n y = X$$

$$f(D) y = X$$

where, $f(D) = D^n + P_1 D^{n-1} + P_2 D^{n-2} + \cdots P_n = 0$

Case(i): when AE has distinct & real roots i.e. $m_1 \neq m_2 \neq m_3$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

Case(ii): when AE has real roots some equal roots

If $m_1 = m_2 \neq m_3 \neq m_4$

$$y = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_3 x} + \cdots$$

Complementary function.

Case(iv): when A.E has complex roots

$m_1, m_2 = \alpha \pm i\beta$; m_3 ; m_4

$$y = (A \cos \beta x + B \sin \beta x) e^{\alpha x} + c_3 e^{m_3 x} + c_4 e^{m_4 x} + \cdots$$

If complex roots are equal

$$y = \left[(A_1 + A_2 x) \cos \beta x + (B_1 + B_2 x) \sin \beta x\right] e^{\alpha x} + c_5 e^{m_5 x} + \cdots$$

Particular Integral:

when the eqn is $f(D)y = X$

the general solution $y = C.F + P.I$

A particular integral is given by $\frac{1}{f(D)}X$

I. when $X = e^{ax}$

$$P.I = \frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax} \quad \text{if } f(a) \neq 0$$

II when $f(a) = 0$

$$P.I = \frac{1}{f(D)}e^{ax} = \frac{x}{f'(a)}e^{ax} \quad \text{if } f'(a) \neq 0$$

III when $f'(a) = 0$

$$P.I = \frac{1}{f(D)}.e^{ax} = \frac{x^2}{f''(a)}e^{ax} \quad \text{if } f''(a) \neq 0$$

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Euler's formula:

Long columns were first analyzed mathematically by the swiss mathematician Leonhard Euler in 1757. He ignored the effect of direct stresses totally and determined critical loads that would cause failure due to buckling only. His analysis is based on certain assumptions

Assumptions

(i) The columns are initially straight

(ii) The columns are made up of homogenous material

(iii) The columns carry perfectly axial loads

(iv) The columns have uniform c/s area throughout

(v) plane c/s of column normal to the centre line remain plane during the phenomenon of buckling i.e., the effect of shear stress is neglected.

(vi) The columns are long compared to the lateral dimensions

(vii) self-wt of column is neglected

(viii) shortening of column due to direct compression is neglected

(ix) The stresses do not exceed the limit of proportionality

(x) Longitudinal fibres of the column are free to expand (or) contract independly without any constraints of adjoining fibres

Case(i): Both ends hinged

Consider a long column (a column that fails due to only buckling) of effective length l, hinged at both ends & crippling under an axial load, called Euler's crippling load P_E. Assume a small lateral force buckles the column. Let the eccentricity at any section 'D' at a height 'x' from 'A' be 'y'. The moment at point is $M = +Py$

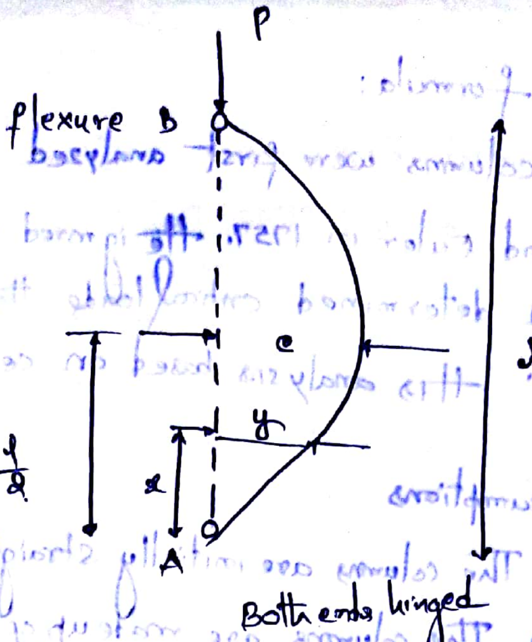
①

~~From double integ~~ differential eqn of flexure

$$EI\frac{d^2y}{dx^2} = -M$$

$$EI\frac{d^2y}{dx^2} = -Py$$

$$EI\frac{d^2y}{dx^2} + Py = 0$$



solving the differential eqn, we have

$$y = c_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + c_2\sin\left(x\sqrt{\frac{P}{EI}}\right)$$

To determine constants of integration, apply end condition at A for which $y=0$; when $x=0$, we have $c_1=0$

And at B for which $y=0$ when $x=l$, we have

$$0 = c_2\sin\left(l\sqrt{\frac{P}{EI}}\right)$$

If c_2 is zero, then the column would not bend at all therefore the other possibility is that

$$\sin\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

$$l\sqrt{\frac{P}{EI}} = 0, \pi, 2\pi, 3\pi \ldots n\pi$$

$$l\sqrt{\frac{P}{EI}} = \pi$$

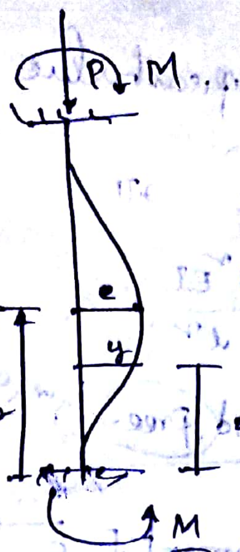
$$l^2\frac{P}{EI} = \pi^2$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}}$$

The crippling load when both ends hinged.

Both ends fixed:

$M = Py - M$



$$-EI\frac{d^2y}{dx^2} = -(Py - M)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{M}{EI}$$

solution of the above DE

$$y = \frac{M}{P} + C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\frac{P}{EI}}\right)$$

$$\frac{\frac{M}{\cancel{EI}}}{D^2 + \frac{P}{\cancel{EI}}} \qquad \frac{M}{P}$$

at the end A where $x = 0$; $\frac{dy}{dx} = 0$

$$C_2\sqrt{\frac{P}{EI}} = 0$$

since 'P' is not zero, therefore C_2

$$\frac{dy}{dx} = 0 - C_1\sin\left(x\sqrt{\frac{P}{EI}}\right)\cdot\sqrt{\frac{P}{EI}} + C_2\cos\left(x\sqrt{\frac{P}{EI}}\right)\cdot\sqrt{\frac{P}{EI}}.$$

put $x = 0$; $\frac{dy}{dx} = 0$ $\qquad P \neq 0$

$$\boxed{C_2 = 0}$$

$x = 0$; $y = 0$ $\qquad \frac{M}{P} + C_1 = 0$

$$C_1 = -\frac{M}{P}$$

$$y = \frac{M}{P} - \frac{M}{P}\cos\left(x\sqrt{\frac{P}{EI}}\right)$$

End conditions $\quad x = l$; $y = 0$.

$$\frac{M}{P} - \frac{M}{P}\cos\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

$$\cos\left(l\sqrt{\frac{P}{EI}}\right) = 1$$

$$l\sqrt{\frac{P}{EI}} = 0, 2\pi, 4\pi \text{ ----}$$

$$l\sqrt{\frac{P}{EI}} = 2\pi$$

Taking the least significant value

$$l\sqrt{\frac{P}{EI}} = 2\pi$$

$$P = \frac{4\pi^2 EI}{l^2}$$

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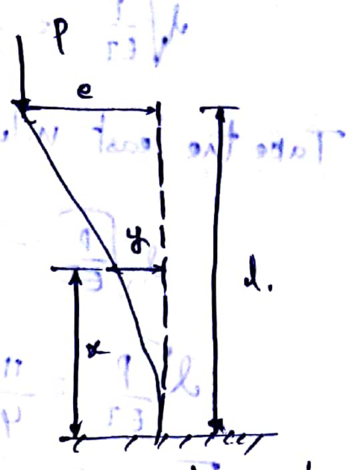
Case(ii):

One end fixed, the other end free

Consider a section at a height 'x' from A having eccentricity 'y'. Let the max. deflection of the free end under crippling load 'p' be e. B.M at the section under consideration is $M = -P(e-y)$



$$EI\frac{d^2y}{dx^2} = P(e-y)$$

$$\frac{Pe}{EI} \cdot e^{0x}$$

$$\therefore \quad \frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P}{EI}e$$

solution of the above differential equation

$$y = e + c_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + c_2\sin\left(x\sqrt{\frac{P}{EI}}\right) \quad \text{---}(1)$$

To know values of constants c_1 & c_2, apply end conditions at A

where $y=0$; $x=0$ $\quad 0 + e + c_1(1) + 0 \Rightarrow \boxed{c_1 = -e}$

$$\frac{\frac{Pe}{EI}e^{0x}}{\left(D^2 + \frac{P}{EI}\right)}$$

put $f(0)=0$.

$$\therefore \quad y = e + (-e)\cos\left(x\sqrt{\frac{P}{EI}}\right) + c_2\sin\left(x\sqrt{\frac{P}{EI}}\right)$$

but at 'x' $\quad \frac{dy}{dx} = 0$; when $x=0$

$$0 = c_2 \sin\sqrt{\frac{P}{EI}}$$

since P cannot be zero, therefore 'c_2' is zero

eq(1) therefore 'y' becomes $y = e - e\cos\left(x\sqrt{\frac{P}{EI}}\right)$

At the end B, when $x = l$; $y = e$

$$e = e - e\cos\left(l\sqrt{\frac{P}{EI}}\right)$$

$$e - e\cos\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

$$\cos\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots (2n-1)\frac{\pi}{2}$$

Take the least value we have,

$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$l^2 \frac{P}{EI} = \frac{\pi^2}{4}$$

$$\Rightarrow \boxed{P = \frac{\pi^2 EI}{4l^2}}$$

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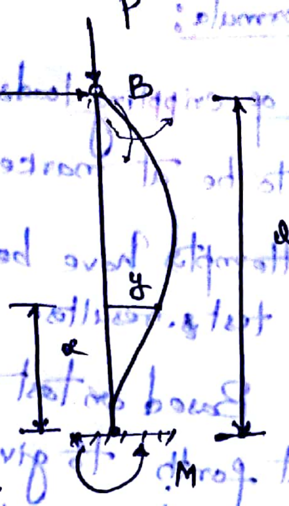
Case iv: One end fixed, other end Hinged.



$$M = +Py - F(l-x)$$

$$EI\frac{d^2y}{dx^2} = -Py + F(l-x)$$

$$EI\frac{d^2y}{dx^2} + Py = F(l-x)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{Fl}{EI} - \frac{Fx}{EI}$$

Case(iv): P.I

$$P.I = \frac{1}{D}[f(D)]^{-1} x^m$$

Auxiliary equ $D^2 + \frac{P}{EI} = 0$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$$

$$D = \pm i\sqrt{\frac{P}{EI}}$$

$$C.F = c_1 \cos\sqrt{\frac{P}{EI}}\,x + c_2 \sin\sqrt{\frac{P}{EI}}\,x$$

$$P.I = \frac{1}{\left(D^2+\frac{P}{EI}\right)}\frac{Fl}{EI}e^{0x} - \frac{1}{\left(D^2+\frac{P}{EI}\right)}\cdot\frac{Fx}{EI}e^{0x}$$

$$\frac{1}{\frac{P}{EI}\left(D^2\frac{EI}{P}+1\right)}\cdot\frac{Fx}{EI}$$

$$\frac{F}{P}\left(1+\frac{EI}{P}D^2\right)^{-1}x$$

$$\frac{F}{P}\left(1 - \frac{EI}{P}D^2 + \frac{E^2I^2}{P^2}D^4\right)x$$

$$\frac{F}{P}(x - 0 + 0)$$

$$\cancel{= \frac{1}{\left(0+\frac{P}{EI}\right)}\cdot\frac{Fl}{EI} - \frac{1}{D\left(D+\frac{P}{EI}\frac{1}{D}\right)}\cdot\frac{Fx}{EI}}$$

$$\cancel{= \frac{Fl}{P} - \frac{1}{D^2}(\;)}$$

$$P.I = \frac{Fl}{P} - \frac{Fx}{P}$$

$$= \frac{F}{P}(l-x) \checkmark$$

$$\therefore y = c_1\cos\sqrt{\frac{P}{EI}}\,x + c_2\sin\sqrt{\frac{P}{EI}}\,x + \frac{F}{P}(l-x)$$

Rankine's formula:

The values of crippling loads for columns given by Euler's formula have been found to be at marked variance with the laboratory results.

Attempts have been made to bridge the gulf between calculated laboratory test results.

Based on test results, some empirical formulae have been put forth to give average stress at failure for different values of slenderness ratios. Obviously, each empirical formula is subject to certain limitations of either slenderness ratio or the end conditions.

We plan to study here the following formulae:

(i) Rankine's formula
(ii) straight line formula
(iii) IS code formula

(i) Rankine's formula:

A short column shall fail at a load $P = \sigma_c \times A$ where σ_c is the crushing stress for the column material but, A column that is long enough to fall in the purview of Euler's formula shall buckle at the crippling load $P_e = \frac{\pi^2 EI}{l^2}$. Mostly the struts & columns are such that they fall neither in the first category nor in the second.

Rankine proposed the following empirical relationship to cover all cases from a short to a very long column.

$$\frac{1}{P_r} = \frac{1}{P} + \frac{1}{P_c}$$

P_r = Rankine's crippling load for the column

P = ultimate load for a short column $= \sigma_c \times A$.

P_c = Euler's crippling load for a long column $= \dfrac{\pi^2 EI}{l^2}$

For a given column, $P = \sigma_c \times A$ is constant & so is $\dfrac{1}{P}$

Euler's crippling load $P_c = \dfrac{\pi^2 EI}{l^2}$ shall increase with decrease in length & $\dfrac{1}{P_c}$ shall be very small. so $\dfrac{1}{P_c}$ decreases with decrease in length of column whereas $\dfrac{1}{P}$ remains constant. Thus, for a short Column $\dfrac{1}{P_r} \rightarrow \dfrac{1}{P}$ or $P_r \rightarrow P$. If, however, the column is long then P_c shall be small & $\dfrac{1}{P}$ with increase in length becomes negligible compared to $\dfrac{1}{P_c}$. In that case, $\dfrac{1}{P_r} \rightarrow \dfrac{1}{P_c}$

$P_r \longrightarrow P_c$

Rankine's formula can be arranged as

$$\frac{1}{P_r} = \frac{1}{P} + \frac{1}{P_c}$$

$$P_r = \frac{P_c + P}{PP_c} = \frac{P}{1 + \frac{P}{P_c}} = \frac{\sigma_c \times A}{1 + \frac{\sigma_c A l^2}{\pi^2 E I}}$$

$$= \frac{\sigma_c A}{1 + \frac{\sigma_c \times A l^2}{\pi^2 E \times A k^2}}$$

$\left[k = \sqrt{\dfrac{I}{A}} \right.$

$k^2 = \dfrac{I}{A} \Rightarrow I = Ak^2$

$$= \frac{\sigma_c A}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{l}{k}\right)^2}$$

For a given material, $\dfrac{\sigma_c}{\pi^2 E}$ is a constant, Replacing it by the constant (a) in the above relation, we have

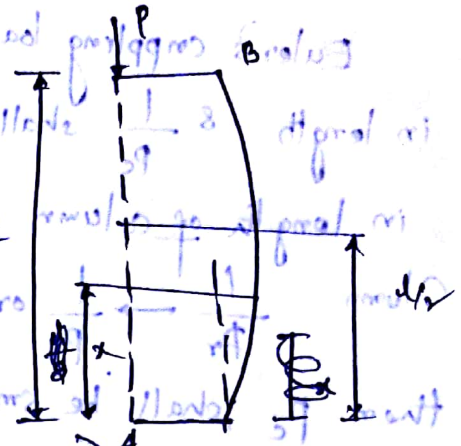
$$P = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{k}\right)^2}$$

a is Rankine's constant

Eccentric Loading for long columns.

Consider a section at a height x from the end A of a column of length 'l' hinged at both ends carrying a load P at an initial eccentricity 'e'. Obviously, the bottom reaction is P. At the section, we have,

$M = P(e+y)$



$$EI\frac{d^2y}{dx^2} = -P(e+y)$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = -\frac{Pe}{EI}$$

solution of the above DE

$$y = c_1\cos\left(\sqrt{\frac{P}{EI}}x\right) + c_2\sin\left(\sqrt{\frac{P}{EI}}x\right) - e$$

Let $\sqrt{\frac{P}{EI}} = \mu$

$$y = c_1\cos\mu x + c_2\sin\mu x - e$$

To determine the constants c_1 & c_2, apply end conditions at A

$x = 0;\ y = 0$ $\quad 0 = c_1 - e$ or $c_1 = e$.

$$y = e\cos\mu x + c_2\sin\mu x - e$$

$$\frac{dy}{dx} = -e\mu\sin\mu x + c_2\mu\cos\mu x$$

At mid-height C; where $x = \frac{l}{2}$; $\frac{dy}{dx} = 0$

$$0 = -e\mu\sin\left(\frac{\mu l}{2}\right) + c_2\mu\cos\frac{\mu l}{2}$$

$$c_2 = e\tan\frac{\mu l}{2}$$

$$\therefore y = e\cos\mu x + e\tan\frac{\mu l}{2}\sin\mu x - e$$

Obviously, the deflection shall be max, at mid height C where $x = \frac{l}{2}$

$$y_{max} = e\cos\frac{\mu l}{2} + e\tan\frac{\mu l}{2}\sin\frac{\mu l}{2} - e$$

$$y_{max} = \frac{e\cos^2\frac{\mu l}{2} + e\sin^2\frac{\mu l}{2}}{\cos\frac{\mu l}{2}} - e$$

$$= e\cdot\sec\frac{\mu l}{2} - e$$

Max B.M is

$$M_{max} = P(e + y_{max})$$

$$= Pe\sec\frac{\mu l}{2}.$$

Max bending stress

$$\sigma_{max}(\text{bending}) = M_{max}\times\frac{y}{I}$$

$$= Pe\sec\frac{\mu l}{2}\times\frac{y}{Ak^2}$$

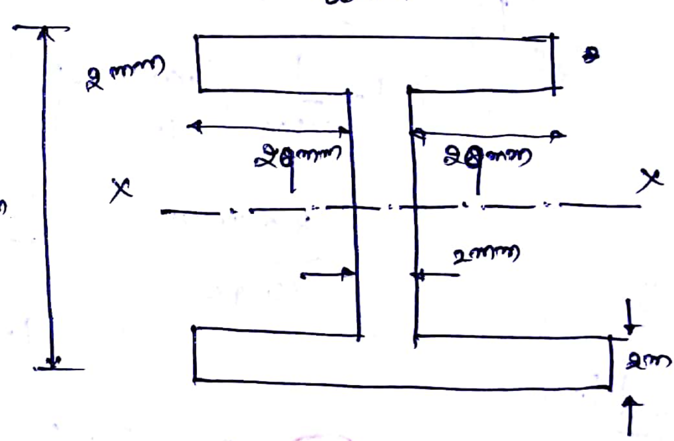
Max stress is

$$\sigma_{max} = \sigma_d + \sigma_{b\,max}$$

$$= \frac{P}{A} + Pe\sec\frac{\mu l}{2}\times\frac{y}{Ak^2}$$

Rankine's formulae:

Compare the buckling loads given by Euler's & Rankine formulae for a mild steel column of an I-section of 400cm length & having pin-joints at both ends. Take yield stress as 310 N mm^2; Rankine's constant $a = \frac{1}{7500}$ & E = 200 kN/mm^2



Sol: Buckling load of a given ~~ss~~ mild steel column ~~when~~ if both ends hinged.

$$P = \frac{\pi^2 EI}{l_e^2}$$

$$\left[l_e = \frac{l}{K} \quad \text{both ends hinged} \quad K = 1 \right.$$

$$E = 200 \text{ kN/mm}^2$$

The given I-section is symmetry

$$I_{xx} = \frac{60 \times 120^3}{12} - 2\left(\frac{29 \times 116^3}{12}\right)$$

$$= \underline{1095669.33} \text{ mm}^4$$

$$I_{yy} = \frac{2\times60^3}{12} + \frac{116\times2^3}{12} + \frac{2\times60^3}{12}$$

$$= 72077.33\ mm^4.$$

From above I_{xx} & I_{yy} values which is minimum.

$$P_c = \frac{\pi^2\times(200\times10^3)\times(7.21\times10^4)}{(400\times10)^2} = 8.895\times10^3\ N$$

1.57×10^{-4}

1.33×10^{-4}

$$\boxed{P_c = 8.895\ kN}$$

$$P_r = \frac{P P_c}{P_c + P}$$

$$= \frac{P\cdot \cancel{P_c}}{\cancel{P_c}\left(1+\frac{P}{P_c}\right)}$$

$$= \frac{\sigma_c\times A}{\left(1+\frac{\sigma_c\times A\cdot l^2}{\pi^2 E I}\right)}$$

Ak^2

$$= \frac{\sigma_c A}{\left(1+\frac{\sigma_c}{\pi^2 E}\left(\frac{l}{k}\right)^2\right)}$$

$$P_r = \frac{\sigma_c\times A}{\left(1+a\left(\frac{l}{k}\right)^2\right)}$$

$$a = \frac{\sigma_c}{\pi^2 E} = \frac{1}{7500}$$

$$\Rightarrow a = \frac{310\ N/mm^2}{\pi^2\times2\times10^5\ N/mm^2} = \frac{1}{6369.42} = \frac{1}{6370}\ ✓$$

$$P_r = \frac{310\times472}{\left(1+\frac{1}{7500}\left(\frac{400\times10}{12.36}\right)^2\right)}$$

$$A = 2(60\times2) + (116\times2)$$

$$= 240 + 232$$

$$= 472\ mm^2$$

$$= \frac{310\times472}{14.96} = 9780.74\ N$$

$$= 9.78\ kN.$$

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From eccentric loading

$$M_{max} = P(y_{max} + e)$$

$$= P(\cancel{e} + e\sec(\sqrt{\tfrac{P}{EI}} \cdot \tfrac{l}{2}) - \cancel{e})$$

$$= Pe \sec(\sqrt{\tfrac{P}{EI}} \cdot \tfrac{l}{2})$$

Max, stresses

$$= \sigma_b + \sigma_d$$

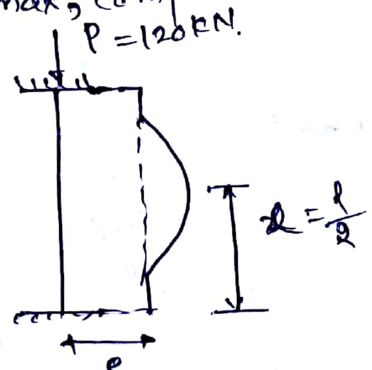
$$= \frac{Pe\sec(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2}) \cdot y}{I} + \frac{P}{A} = \frac{Pe\sec(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2})y}{Ak^2} + \frac{P}{A}$$

$$\text{Max, stresses} = \frac{P}{A}\left(1 + \frac{e\sec(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2})y}{k^2}\right)$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{e\sec(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2})y}{k^2}\right)$$

$$\boxed{P = \frac{A\,\sigma_{max}}{\left[1 + \left[\frac{e\sec(\sqrt{\frac{P}{EI}} \cdot \frac{l}{2})y}{k^2}\right]\right]}}$$

EX.01: A column of circular section has 150 mm ϕ & 3 m length. Both ends of the column are fixed. The column carries a load of 120 kN at eccentricity of 15 mm from the geometrical axis of the columns. Find the max, compressive stress on the column section.



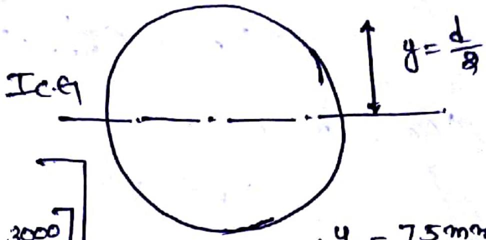
Sol: From above ~~formula~~ problem ends are fixed & but max compressive stress are known when both ends hinged, whereas similar end conditions so, it is ~~to~~ applicable to the problem

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{e y \sec\frac{\mu l}{2}}{k^2}\right)$$

$P = 120\,kN = 120\times10^3\,N$; $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times150^2 = 17671.45\,mm^2$

$$I_{c.g} = \frac{\pi d^4}{64} = \frac{\pi\times150^4}{64}$$

$$= 2.48\times10^7\,mm^4$$



$y = 75\,mm$

$$\sigma_{max} = \frac{120\times10^3}{17671.45}\left[1 + \frac{15\times75\times \sec\left[1.56\times10^{-4}\times\frac{3000}{2}\right]}{1403.39}\right]$$

$$k = \sqrt{\frac{2.48\times10^7}{17671.45}}$$

$$k^2 = 1403.39$$

$$\mu = \sqrt{\frac{P}{EI}}$$

$$= \sqrt{\frac{120\times10^3\,N}{2\times10^5\times2.48\times10^7\,N\text{-}mm^2}}$$

$E = 2\times10^5\,N/mm^2$

$\mu = 1.56\times10^{-4}$

$$\sec\left[1.56\times10^{-4}\times\frac{3000}{2}\right]$$

$\sec(13.4^\circ)$

$$\sigma_{max} = \frac{120\times10^3}{17671.45}\left[1 + \frac{1156.48}{1403.39}\right]$$

$$= \frac{120\times10^3}{17671.45}(1.82)$$

$\sigma_{max} = \cancel{0.679\,N/mm^2}$

$$\boxed{\sigma_{max} = 1.23\,N/mm^2}$$

let the max, eccentricity be e' for the tension not to develop. Now $(\sigma_d - \sigma_{b\,max})$ must not be $-$ve.

$$\sigma_d \geq \sigma_{b\,max}$$

$$0.679\,N/mm^2 \geq 1.23\,N/mm^2$$

Radius of Gyration $K = \sqrt{\frac{I_{xx}}{A}}$

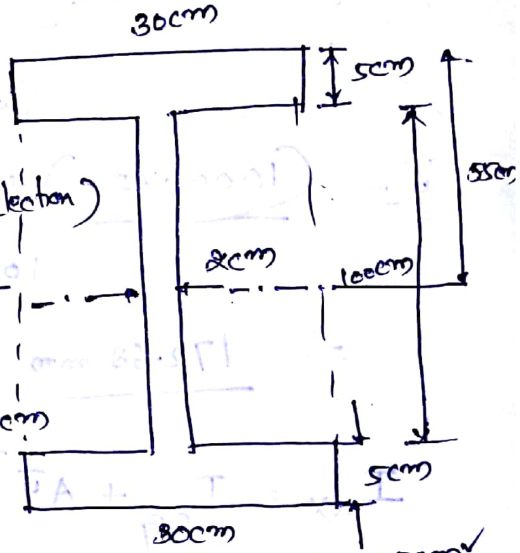
$I_{xx} = 4.74 \times 10^8\ mm^4$

$A = 33750\ mm^2$

$K = \sqrt{\frac{4.74 \times 10^8}{33750}} = 118.50\ mm$

Ex 02: A built-up beam shown in fig. is simply supported at ends. compute its length, given that when it is subjected to a load 40KN per metre length, it deflects by 1 cm.. Find out the safe load if this beam is used in a column with both ends fixed. Assume FOS 4. Use Euler's formula

$E = 21\ MN/cm^2$



Sol: From the given data

beam is deflected by 1cm (Assume it is max, deflection)

Max. deflection of S.S.B when the load is UDL

w KN/m (or) w N/cm

$y_{max} = \frac{5wl^4}{384EI}$

w – load intensity N/cm

l in cm

$w = 40\ KN/m = 40 \times 10^3\ N/m$

$= 40 \times 10^3 \times 10^{-2}\ N/cm$

$= 400\ N/cm$

$\bar{y} = 55cm$

$I_{C\text{-}G} = \frac{30 \times 110^3}{12} - 2\left[\frac{14 \times 100^3}{12}\right]$

$= 994166.67\ cm^4$

$\therefore 1\ cm = \frac{5 \times 400 \times l^4}{384 \times (21 \times 10^6) \times 994166.67}$

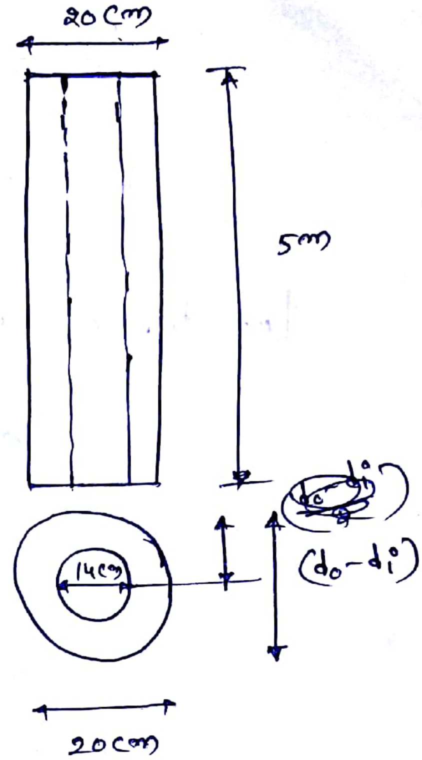
$\Rightarrow l = 1414.96 \simeq 1415\ cm$

length of the beam $l = 1415\ cm$

when this beam is used as column with both ends fixed. we have

effective length $l_e = \frac{l}{2} = \frac{1415}{2} = 707.5\ cm$

1. A hollow circular column of 5m length, 20cm external diameter & 14cm internal diameter is fixed at both ends. It carries a load of 200 kN at an eccentricity of 1.5cm from the axis of the column. Determine the max, stresses developed. what should be limiting eccentricity if tension is not to develop. Take $E = 9500\ kN/cm^2$



Sol: Here, end condition is both ends fixed but max stress developed in a column is known when both ends hinged, ~~but~~ However, stresses developed in a column for these conditions are same.

$$\therefore \sigma_{max} = \sigma_d + \sigma_b$$

$$= \frac{P}{A} + \frac{M_{max}\, y_o}{I}$$

$$= \frac{P}{A} + \frac{P e \sec\frac{\mu l}{2} \cdot y_o}{A k^2}$$

$$= \frac{P}{A}\left(1 + \frac{e \sec\frac{\mu l}{2}\, y_c}{k^2}\right)$$

$P = 200\ kN$; $A = \frac{\pi}{4}(d_o)^2 - \frac{\pi}{4} d_i^2$ $\quad d_o = 20\ cm$, $d_i = 14\ cm$

$$A = \frac{\pi}{4}(20)^2 - \frac{\pi}{4}(14)^2 = 160.22\ cm^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(20^4 - 14^4) = \cancel{9549.85}\ 5968.24\ cm^4$$

$e = 1.5\ cm$ $\quad l = 500\ cm$

$$\mu = \sqrt{\frac{P}{EI}} = \sqrt{\frac{200}{9500 \times \cancel{9549.85}\ 5968.24}} = \cancel{4.69 \times 10^{-4}}$$

$$y_c = \frac{d_o}{2} = \frac{20}{2} = 10\ cm$$

$$\frac{\mu l}{2} = 1.88 \times 10^{-3} \times \frac{500}{2} = 0.469 \times \frac{180}{\pi}$$

$26.9°$

1.121

$$\frac{\mu l}{2} = \cancel{4.69 \times 10^{-4} \times \frac{500}{2}} = 1.17 \times 10^{-3} \times 10\ rad$$

$$= 1.17 \times 10^{-3} \times 10 \times \frac{180}{\pi} = \cancel{0.067°}\ 6.70°$$

$$\Rightarrow \sec 6.70° = \frac{1}{\cos 6.70°} = 1.0068$$

Maximum compressive stress.

$$\sigma_{max} = \sigma_d + \sigma_{b\,max}$$

$$= \frac{P}{A} + \frac{Pe \sec \frac{\mu l}{2} \cdot y_c}{Ak^2}$$

$$= \frac{200 \times 10^3}{160.22} + \frac{200 \times 10^3 \times 1.5 \times \cancel{1.007}\,1.121 \times 10}{\cancel{160.22} \cdot \cancel{95491.85}\ 5968.24}$$

$$= 1248.28 + 608.22 \ N/cm^2$$

let the max, eccentricity be 'e' for the tension not to develop

$\sigma_d - \sigma_{b\,max}$ ~~must~~ not (-)ve.

$$1248.28 \geq 405.47e$$

$$\Rightarrow e \leq \frac{1248.28}{405.47} \Rightarrow e \leq 3.08$$

The limiting eccentricity for the tension not to develop

is 3.08 cm

24/1/20