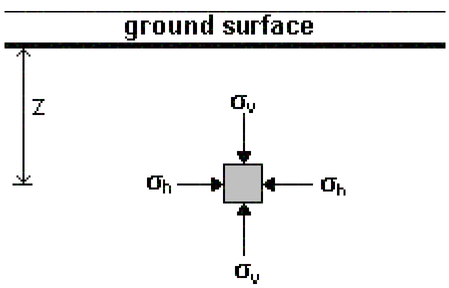
Unit - 4

PRINCIPLE OF EFFECTIVE STRESS

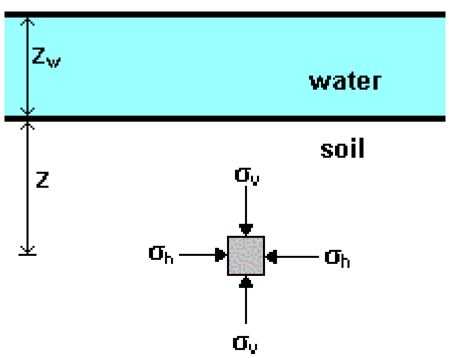
The **principle of effective stress** was enunciated by **Karl Terzaghi** in the year 1936. This principle is **valid only for saturated soils.**

Total Stress (σ)
When a load is applied to soil, it is carried by the solid grains and the water in the pores. The **total vertical stress** acting at a point below the ground surface is due to the weight of everything that lies above, including soil, water, and surface loading. Total stress thus increases with depth and with unit weight. Total stress is parameter which can be computed or even measured with suitable instruments such as a pressure cell.

Vertical total stress at depth z, $\boldsymbol{\sigma_v = \gamma.Z}$



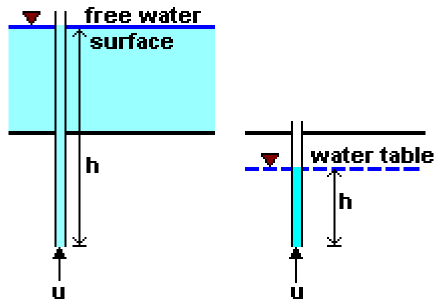
Below a water body, the total stress is the sum of the weight of the soil up to the surface and the weight of water above this. $\boldsymbol{\sigma_v = \gamma.Z + \gamma_w.Z_w}$



The total stress may also be denoted by σ_z or just $\boldsymbol{\sigma}$**.** It varies with changes in water level and with excavation.

Pore Water Pressure (u)
The pressure of water in the pores of the soil is called **pore water pressure (u).** The magnitude of pore water pressure depends on:

- the depth below the water table.
- the conditions of seepage flow.



Pore water pressure at any point is simply equal to the depth (h) of the point below the groundwater table. Under hydrostatic conditions, no flow takes place, and the pore pressure at a given point is given by

$$\mathbf{u = \gamma_w . h}$$

where h = depth below water table or overlying water surface

It is convenient to think of pore water pressure as the pressure exerted by a column of water in an imaginary standpipe inserted at the given point. A stand pipe or a piezometer is used to measure the pore water pressure.

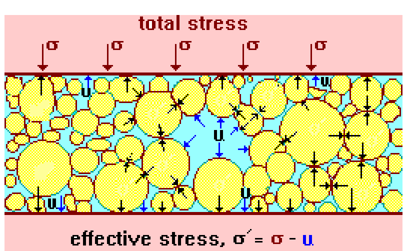
The natural level of ground water is called the **water table** or the **phreatic surface.** Under conditions of no seepage flow, the water table is horizontal. The magnitude of the pore water pressure at the water table is zero. Below the water table, pore water pressures are positive.

Effective Stress ($\overline{\sigma}$ or σ')

At any point in a soil mass, the effective stress is related to total stress (σ) and pore water pressure (u) as

$$\overline{\sigma} = \sigma - u$$

Both the total stress and pore water pressure can be measured at any point. All measurable effects of a change of stress, such as compression and a change of shearing resistance, are exclusively due to changes in effective stress.



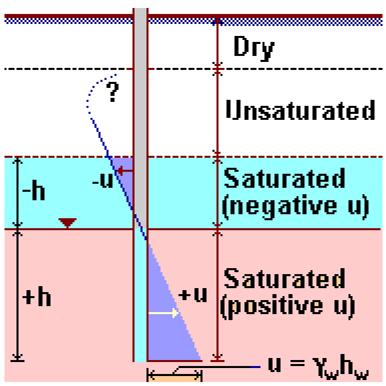
In a saturated soil system, as the voids are completely filled with water, the pore water pressure acts equally in all directions. Hence, pore water pressure is also called **neutral stress**. It has no shear stress component.

The effective stress is not the exact contact stress between particles but the distribution of load carried by the soil particles over the area considered. It cannot be measured and can only be computed.

If the total stress is increased due to additional load applied to the soil, the pore water pressure initially increases to counteract the additional stress. The entire additional load is initially borne by the pore water pressure. This increase in pressure within the pores might cause water to drain out of the soil mass, and the load is gradually transferred to the solid grains. This will lead to decrease in pore water pressure and increase of effective stress.

EFFECTIVE STRESS IN UNSATURATED ZONE

Above the water table, when the soil is saturated, pore pressure will be negative (less than atmospheric). The height above the water table to which the soil is saturated is called the **capillary rise**, and this depends on the grain size and the size of pores. In coarse soils, the capillary rise is very small.



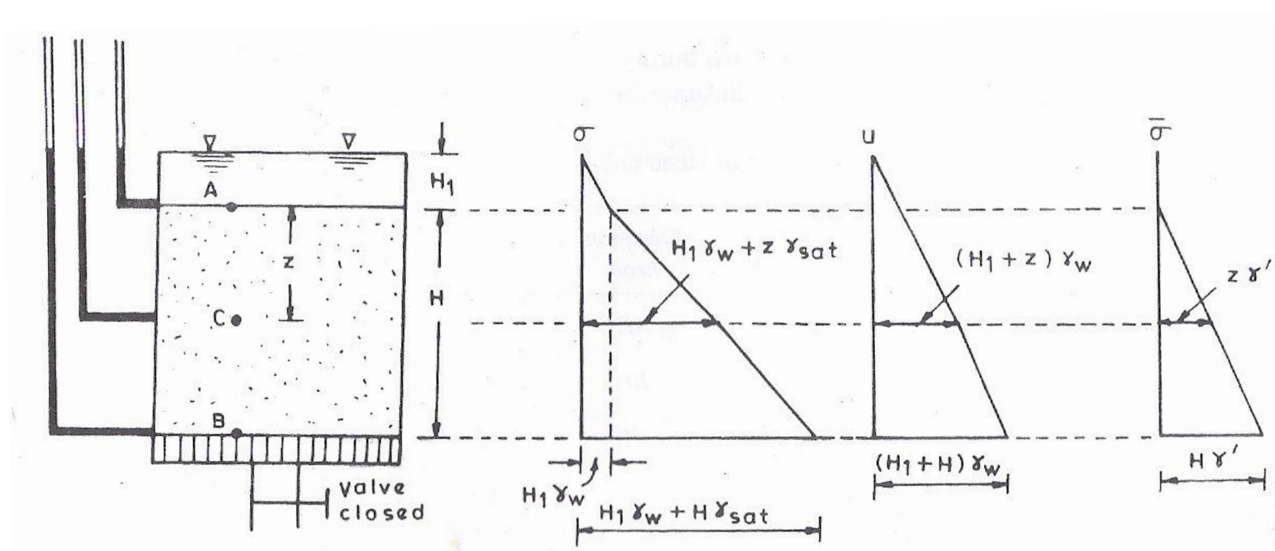
Between the top of the saturated zone and the ground surface, the soil is partially saturated, with a consequent reduction in unit weight . The pore pressure in a partially saturated soil consists of two components:

Pore water pressure = u_w

Pore air pressure = u_a

Water is incompressible, whereas air is compressible. The combined effect is a complex relationship involving partial pressures and the degree of saturation of the soil.

EFFECTIVE STRESS UNDER NO FLOW CONDITION

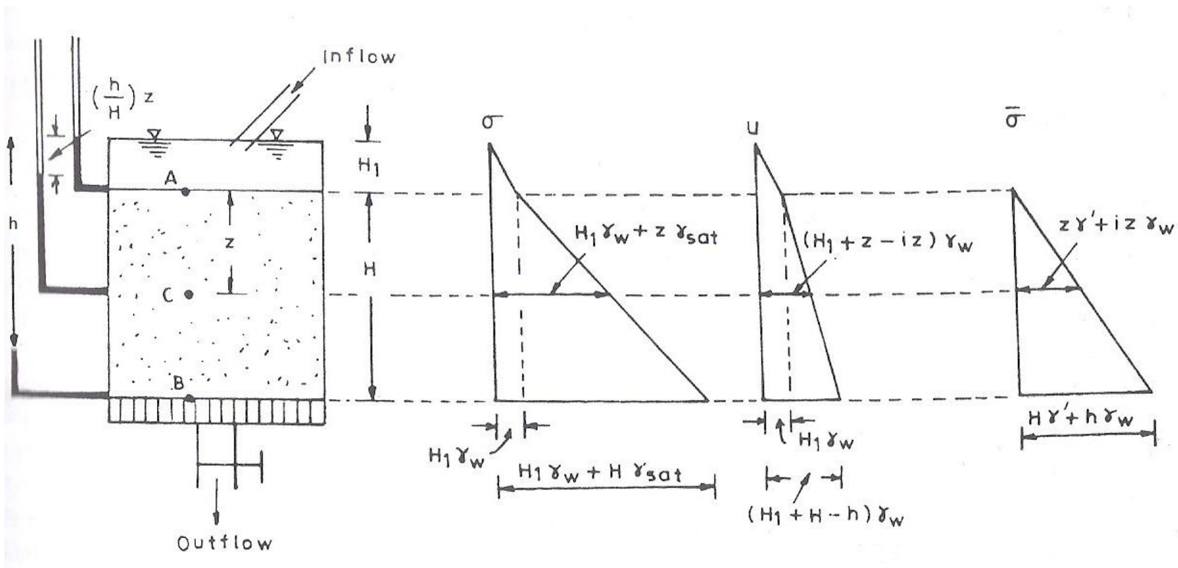


Variation of σ, u and σ̄ with depth for 'no flow condition

When no flow is taking place in the soil, the condition is known as no-flow condition. Figure above shows a tank filled with submerged soil, with no seepage occurring, because the valve at the bottom is closed. Hence, the water levels in the standpipes inserted at the top, bottom and any intermediate position of the soil layer are the same.

At A:	**At B:**
Total stress, $\sigma_A = H_1\gamma_w$ Pore water pressure, $u_A = H_1\gamma_w$ Effective stress, $\overline{\sigma}_A = \sigma_A - u_A = 0$	Total stress, $\sigma_B = H\gamma_{sat} + H_1\gamma_w$ Pore water pressure, $u_B = (H + H_1)\gamma_w$ Effective stress, $\overline{\sigma}_B = \sigma_B - u_B = H\gamma_{sat} + H_1\gamma_w - (H + H_1)\gamma_w$ $or\ \overline{\sigma}_B = H(\gamma_{sat} - \gamma_w) = H\gamma'$

EFFECTIVE STRESS UNDER DOWNWARD FLOW CONDITION



Variation of σ, u and $\overline{\sigma}$ with depth for downward flow

When water flows through the soil, it exerts drag forces called seepage forces on the individual grains of soil. **Seepage forces act in the direction of flow.** The presence of seepage forces will cause changes in the pore water pressures and effective stresses in the soil. Figure above shows the variation of total stress, pore water pressure and effective stress with depth for down ward flow condition.

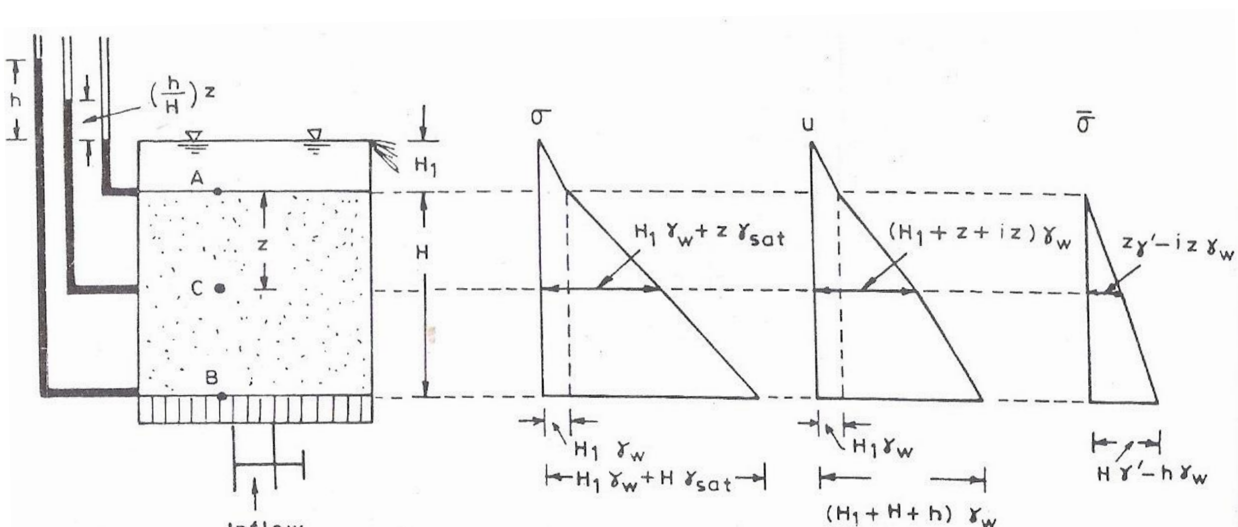
At A:	**At B:**
Total stress, $\sigma_A = H_1\gamma_w$ Pore water pressure, $u_A = H_1\gamma_w$ Effective stress, $\overline{\sigma}_A = \sigma_A - u_A = 0$	Total stress, $\sigma_B = H\gamma_{sat} + H_1\gamma_w$ Pore water pressure, $u_B = (H + H_1 - h)\gamma_w$ Effective stress, $\overline{\sigma}_B = \sigma_B - u_B = H\gamma_{sat} + H_1\gamma_w - (H + H_1 - h)\gamma_w$ $or\ \overline{\sigma}_B = H(\gamma_{sat} - \gamma_w) + h\gamma_w = H\gamma' + h\gamma_w$

For no flow condition, the effective stress at B is $H\gamma'$ whereas for downward flow condition the effective stress at B is $H\gamma' + h\gamma_w$. Hence, the effective stress increases by $h\gamma_w$ when there is a downward flow. **Thus, downward seepage means an increase in effective stress.** This additional stress is due to the frictional force or drag acting on the surface of grains which form the walls of the pores.

Over a length of flow H, the head loss is h. Therefore, over a length of flow z, the head loss is $\left(\frac{h}{H}\right)z$. Therefore, at point C, at a depth z below the soil surface, the increase in effective stress is $\left(\frac{h}{H}\right)z\gamma_w \ or \ iz\gamma_w$. Seepage pressure is thus equal to $iz\gamma_w$.

At C:
Total stress, $\sigma_C = z\gamma_{sat} + H_1\gamma_w$
Pore water pressure, $u_C = \left[z + H_1 - \left(\frac{h}{H}\right)z\right]\gamma_w = z + H_1 - \left(\frac{h}{H}\right)z$
Effective stress,
$\overline{\sigma}_C = \sigma_C - u_C = z\gamma_{sat} + H_1\gamma_w - \left(z + H_1 - \left(\frac{h}{H}\right)z\right)\gamma_w$
$or \ \overline{\sigma}_C = z\gamma' + iz\gamma_w$

EFFECTIVE STRESS UNDER UPWARD FLOW CONDITION



Variation of σ, u and $\overline{\sigma}$ with depth for upward flow

The figure above shows the condition of upward seepage. It is caused by opening the valve located below the tank.

At A:	**At B:**
Total stress, $\sigma_A = H_1\gamma_w$ Pore water pressure, $u_A = H_1\gamma_w$ Effective stress, $\overline{\sigma}_A = \sigma_A - u_A = 0$	Total stress, $\sigma_B = H\gamma_{sat} + H_1\gamma_w$ Pore water pressure, $u_B = (H + H_1 + h)\gamma_w$ Effective stress, $\overline{\sigma}_B = \sigma_B - u_B = H\gamma_{sat} + H_1\gamma_w - (H + H_1 + h)\gamma_w$ $or \ \overline{\sigma}_B = H(\gamma_{sat} - \gamma_w) - h\gamma_w = H\gamma' - h\gamma_w$ $or \ \overline{\sigma}_B = H\gamma' - \left(\frac{h}{H}\right)H\gamma' = H\gamma' - iH\gamma_w$

In the case of upward flow, seepage pressure acts in the upward direction, and reduces the effective stress at the level of point B by $iH\gamma_w$.

QUICK SAND CONDITION

For the upward flow condition, the effective stress at the level of point B is given by,

$$\bar{\sigma}_B = H\gamma' - iH\gamma_w$$

When the effective stress is reduced to zero, the above expression may be written as

$$H\gamma' - iH\gamma_w = 0$$

$$or\ H\gamma' = iH\gamma_w$$

here, the seepage pressure becomes equal to the effective pressure so that the effective stress throughout the soil is reduced to zero.

$$or \quad i = i_{cr} = \frac{\gamma'}{\gamma_w}$$

i_{cr} is called the *critical hydraulic gradient*. When upward flow takes place the critical hydraulic gradient, a soil such as coarse silt or fine sand looses all its shearing strength and it cannot support any load. The soil is said to have become 'quick' or 'alive' and *boiling* will occur. In such a situation, effective stress is reduced to zero and the soil behaves like a very viscous liquid. Such a state is known as **quick sand condition.**

If the void ratio of the natural deposit is known, i_{cr} can be computed as follows:

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$

$$\gamma_{sat} - \gamma_w = \left(\frac{G+e}{1+e}\right)\gamma_w - \gamma_w$$

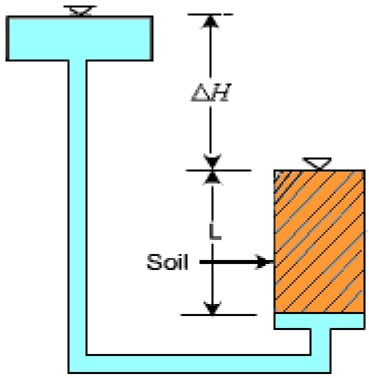
$$\gamma' = \left(\frac{G-1}{1+e}\right)\gamma_w$$

$$or \quad i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{G-1}{1+e}$$

Assuming G = 2.65, i_{cr} will vary from 1.1 for e = 0.5 (dense sand) to 0.83 for e =1 (loose sand). For usual void ratios in sand soils of 0.6 to 0.7, the critical hydraulic gradient will just be about 1.

Seepage forces affect sands more than clays because sands do not possess cohesion, while fine silts and clays have some inherent cohesion which holds the grains together even at critical hydraulic gradient. Boiling does not occur in coarse sands and gravels either, because these soils are highly pervious and large discharges are required to produce $i_{cr} = 1$.

Alternate approach for critical hydraulic gradient



At the bottom of the soil column,

$\sigma = \gamma_{sat} L$

$u = (L + \Delta H)\gamma_w$

During quick sand condition, the effective stress is reduced to zero.

$\gamma_{sat} L = (L + \Delta H)\gamma_w$

Simplifying, $\frac{\gamma'}{\gamma_w} = i_{cr} = \frac{\Delta H}{L}$

where i_{cr} = **critical hydraulic gradient**

THE IMPORTANCE OF EFFECTIVE STRESS

At any point within the soil mass, the magnitudes of both total stress and pore water pressure are dependent on the ground water position. With a shift in the water table due to seasonal fluctuations, there is a resulting change in the distribution in pore water pressure with depth.

Changes in water level ***below ground*** result in changes in effective stresses below the water table. A rise increases the pore water pressure at all elevations thus causing a decrease in effective stress. In contrast, a fall in the water table produces an increase in the effective stress.

Changes in water level ***above ground*** do not cause changes in effective stresses in the ground below. A rise of water level above ground surface increases both the total stress and the pore water pressure by the same amount, and consequently effective stress is not altered.

In some analyses it is better to work with the *changes* of quantity, rather than in absolute quantities. The effective stress expression then becomes:

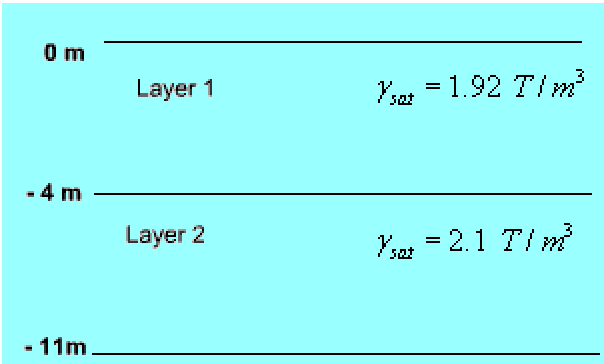
$$\Delta\sigma' = \Delta\sigma - \Delta u$$

If both total stress and pore water pressure change by the same amount, the effective stress remains constant.

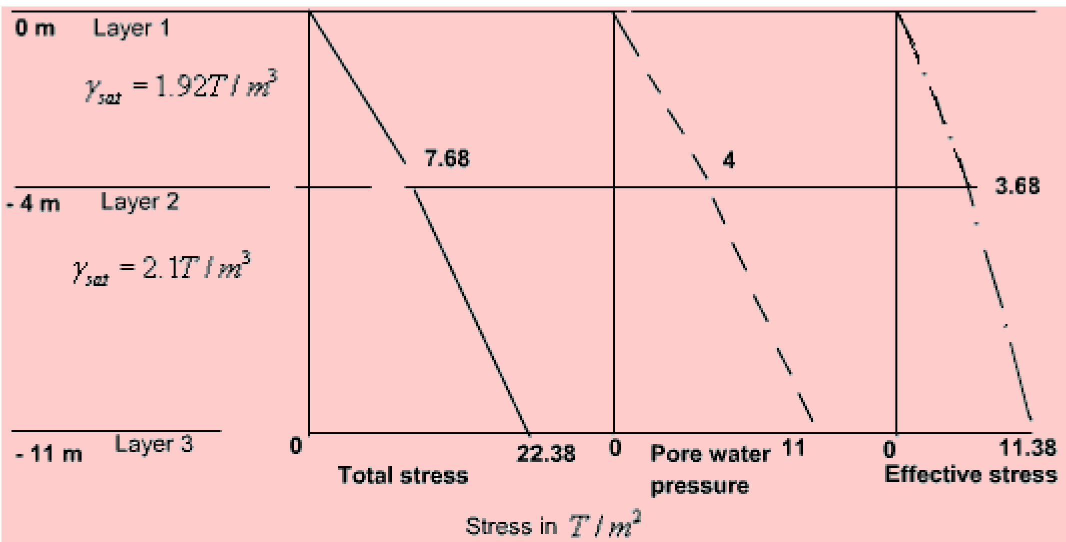
Total and effective stresses must be distinguishable in all calculations. Ground movements and instabilities can be caused by changes in total stress, such as caused by loading by foundations and unloading due to excavations. They can also be caused by changes in pore water pressures, such as failure of slopes after rainfall.

Boiling may occur when excavations are made below water table and water is pumped out from the excavation pit to keep the area free from water. Boiling can be prevented by lowering the water table at the site before excavation or alternately, by increasing the length of upward flow. Boiling condition may also occur when a pervious sand stratum underying a clay soil is in artesian pressure condition.

Example 1: For the soil deposit shown below, draw the total stress, pore water pressure and effective stress diagrams. The water table is at ground level.



Solution:



Total stress

At - 4m, σ = 1.92 x 4 = 7.68 T/m^2

At -11m, σ = 7.68 + 2.1 x 7 = 22.38 T/m^2

Pore water pressure

At - 4 m, u = 1 x 4 = 4 T/m^2

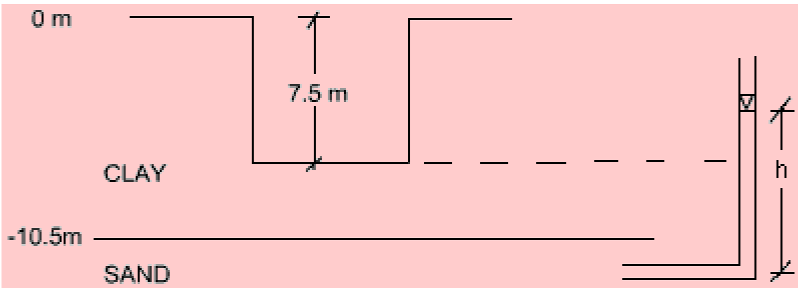
At -11 m, u = 1 x 11 = 11 T/m^2

Effective stress

At - 4 m , $\bar{\sigma}$ = 7.68 - 4 = 3.68 T/m^2

At -11m , $\bar{\sigma}$ = 22.38 - 11 = 11.38 T/m^2

Example 2: An excavation was made in a clay stratum having γ_t = 2 T/m^3. When the depth was 7.5 m, the bottom of the excavation cracked and the pit was filled by a mixture of sand and water. The thickness of the clay layer was 10.5 m, and below it was a layer of pervious water-bearing sand. How much was the artesian pressure in the sand layer?



Solution:

When the depth of excavation was 7.5 m, at the interface of the CLAY and SAND layers, the effective stress was equal to zero.

Downward pressure due to weight of clay = Upward pressure due to artesian pressure

(10.5 - 7.5) γ_t = $\gamma_w . h$, where h = artesian pressure head
3 x 2 = 1 x h

$\therefore$ h = 6 m = 0.6 kg/cm^2 or 6 T/m^2 artesian pressure

COMPACTION OF SOILS

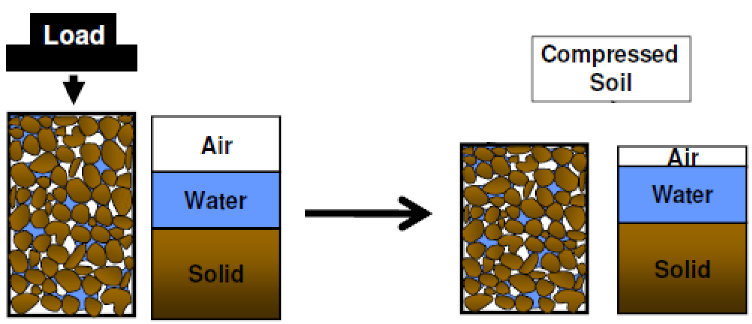
Compaction is the most common and important method of soil improvement. The densification of soil by the expulsion of air and rearrangement of particles, by the application of mechanical energy is known as compaction. .

It is applied to improve the properties of an existing soil or in the process of placing fill such as in the construction of embankments, road bases, runways, earth dams, and reinforced earth walls. Compaction is also used to prepare a level surface during construction of buildings. There is usually no change in the water content and in the size of the individual soil particles.

The objectives of compaction are:

- To increase soil shear strength and therefore its stability and bearing capacity.
- To reduce compressibility and permeability of the soil.
- To prevent detrimental settlements
- To control undesirable volume changes through swelling and shrinkage.
- To increase the stability of slopes and embankments
- To reduce frost damage.
- To reduce erosion damage
- To increase the effective stress.

The degree of compaction of a soil is measured in terms of its dry unit weight, i.e. the amount of osil solids that can be packed in a unit volume of soil.



Laboratory Methods

The compaction characteristics and degree of compaction can be obtained from the laboratory tests. In these tests, a specified amount of compactive effort is applied to a constant volume of soil mass. The compactive energy is reported in J/m^3. Impact compaction is most commonly used.

The variation in compaction with water content and compactive effort is first determined in the laboratory. There are several tests with standard procedures such as:

- Indian Standard Light Compaction Test (similar to Standard Proctor Test)
- Indian Standard Heavy Compaction Test (similar to Modified Proctor Test)

Initial water content

The amount of water to be mixed with air dried soil at the commencement of the test will vary with the type of soil under test. In general, with sandy and gravelly soil, a moisture content of 4 percent to 6 percent would be suitable, while with cohesive soil, a moisture content about 8 percent to 10 percent below the plastic limit of the soil (plastic limit minus 10 to plastic limit minus 8) usually be suitable.

Indian Standard Light compaction Test

6 kg sample passing of air dried soil passing 19 mm sieve shall be taken. Soil is compacted into a 1000 cm^3 mould in 3 equal layers, each layer receiving 25 blows of a 2.6 kg rammer dropped from a height of 310 mm above the soil.

For compacting soil containing coarse material up to 37.5 mm size, the 2250 cc mould should be used. A soil sample weighing 6 kg and passing 37.5 mm IS sieve is used for the test. Soil is compacted in 3 layers, each layer being given 55 blows of the 2.6 kg rammer.

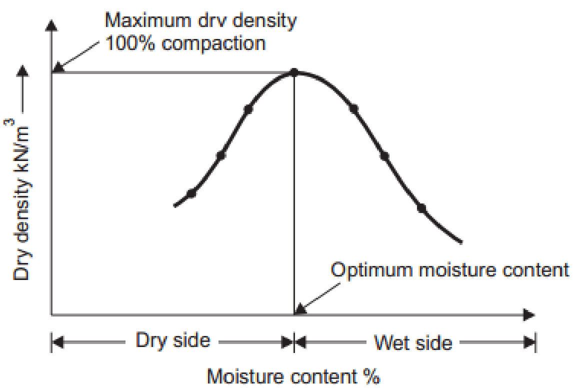
Knowing the wet weight of the compacted soil and its water content, the dry unit weight of the soil can be calculated:

$$\gamma = \frac{Weight\ of\ compacted\ soil}{Volume\ of\ the\ mould}$$

$$\gamma_d = \frac{\gamma}{1+w}$$

The compaction is repeated at various moisture contents. The dry unit weight of each compacted soil sample is plotted against the water content and the curve called compaction curve is obtained. Each data point on the curve represents a single compaction test. Usually four to five points are required to obtain the compaction curve. The inverted V-shaped curve applies only to soils possessing some amount of plasticity.

The compaction curve is unique for a given soil type, method of compaction and compactive effort.



The water content corresponding to the maximum dry unit weight or dry density is known as the **Optimum Moisture Content (OMC).** The maximum dry unit weight so obtained is only for a given amount of compactive effort and the method of compaction. It is not necessarily the maximum dry unit weight that can be obtained in the field.

Compaction energy per unit volume (E)

$$E = \frac{(No.of\ blows\ per\ layer) x\ (No.of\ layers)\ x\ (Wt.of\ hammer)\ x\ (Ht.of\ drop\ of\ hammer)}{Volume\ of\ mould}$$

Indian Standard Heavy Compaction Test
Light Compaction Test (Standard Test) cannot reproduce the densities measured in the field under heavier loading conditions, and this led to the development of the Heavy Compaction Test (Modified Test).

A 5-kg sample of air dried soil passing the 19 mm IS test sieve shall be taken. The equipment and procedure are essentially the same as that used for the Standard Test except that the soil is compacted in 5 layers, each layer also receiving 25 blows. The same mould is also used. To provide the increased compactive effort, a heavier rammer of 4.9 kg and a greater drop height of 450 mm are used.

For compacting soil containing coarse material up to 37.5 mm size, the 2 250 cm^3 mould should be used. A sample weighing about 30 kg and passing the 37.5 mm IS sieve is used for the test. Soil is compacted in five layers, each layer being given 55 blows of the 4.9-kg rammer.

In general, if the percentage of soil retained on 4.75 mm is more than 20%, larger mould of internal dia 150 mm, effective height of 127.3 mm and capacity of 2250 is recommended for both light and heavy compaction.

Factors affecting compaction
Compaction is a function of the following factors:

(i) Water content
(ii) Compactive effort (or amount of compaction)
(iii) Type of soil
(iv) Method of compaction
(v) Admixture

Water Content

As water is added to a soil at low moisture contents, it becomes easier for the particles to move past one another during the application of compacting force. The particles come closer, the voids are reduced and this causes the dry density to increase. As the water content increases, the soil particles develop larger water films around them.

This increase in dry density continues till a stage is reached where water starts occupying the space that could have been occupied by the soil grains. Thus the water at this stage hinders the closer packing of grains and reduces the dry unit weight.

The dry unit weight can also be related to the water content and degree of saturation as

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{G\gamma_w}{1+\frac{wG}{S}}$$

For a given water content, the theoretical maximum value of dry unit weight for a compacted soil is obtained corresponding to the situation when no air voids are left, i.e. when the degree of saturation becomes equal to 100%. This is not the same thing as a soil becoming saturated when its water content is increased so as to fill all the air voids.

The **Zero Air Void Density (ZAVD)** is obtained for a soil at a given water content by substituting S =1 in the equation above.

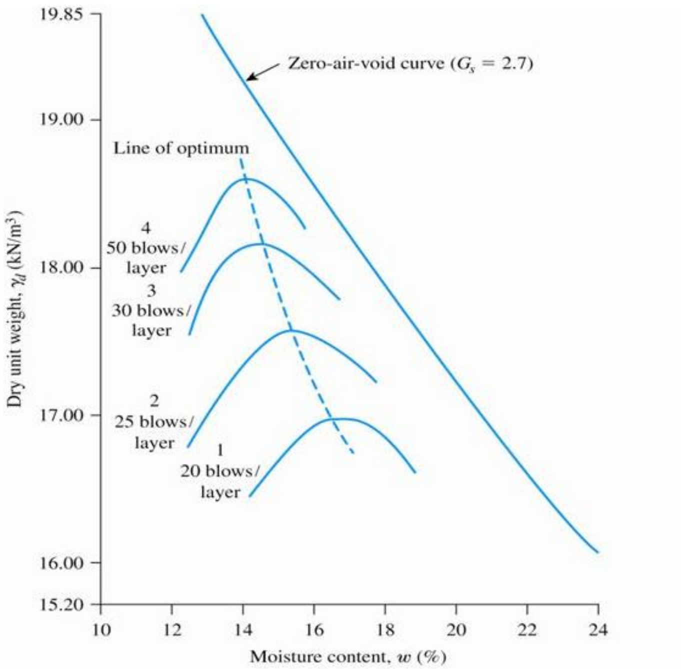
Zero Air Void Density is defined as the maximum dry unit weight that can be ideally obtained for a soil at a given water content by applying compaction.

If ZAVD is calculated for different water content values and plotted alongside the compaction curve, ZAVD curve is obtained.

Compactive effort

For a given type of compaction, the higher the compactive effort, the higher the maximum dry unit weight and lower the OMC. However, as the moulding water content increases, the influence of compaction effort on dry unit weight tends to diminish. Also, the maximum dry unit weight does not go on increasing as the compactive effort is increased. The margin of increase becomes smaller and smaller even on the dry side of the OMC; while on the wet side there is hardly any increase. The degree of saturation at OMC remains almost the same at all compactive efforts.

If the peaks of the compaction curves for different compactive efforts are joined together, **Line of Optimums** is obtained. The line of optimum is nearly parallel to the ZAVD curve.



Type of Soil

1. Coarse grained soils, well graded, compact to high dry unit weights, especially if they contain some fines. However, if the quantity of fines is excessive, maximum dry unit weight decreases.
2. Poorly graded or uniform sands lead to the lowest dry unit weight values.
3. In clay soils, the maximum dry unit weight tends to decrease as plasticity increases.
4. Cohesive soils generally have high values of OMC.

5. Heavy clay clays with high plasticity have low maximum dry unit weight and very high OMC.

Method of Compaction

For the same amount of compactive effort, the dry unit weight will depend upon whether the method compaction utilizes kneading action, dynamic-impact action or static action. Different methods of compaction give their own compaction curves.

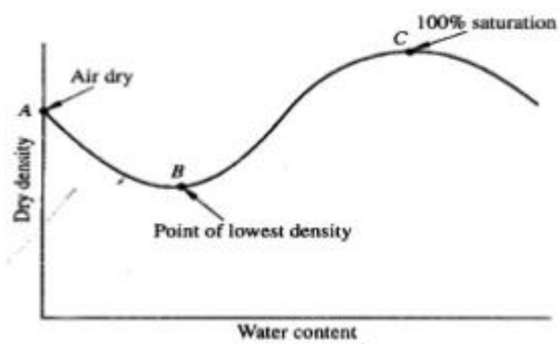
Since the field compaction is essentially a kneading type compaction or rolling type compaction and the laboratory tests use the dynamic-impact type compaction, some divergence in the OMC and MDD values must be expected in the two cases.

Admixture

The compaction characteristics of soils are improved by the addition of admixtures. The most commonly used admixtures are lime, cement and bitumen. The dry density achieved depends on the type and amount of admixtures.

Compaction of Cohesionless Soils

For **cohesionless soils** (or soils without any fines), the standard compaction tests are difficult to perform. For compaction, application of vibrations is the most effective method. Watering is another method. The seepage force of water percolating through a cohesionless soil makes the soil grains occupy a more stable position. However a large quantity of water is required in this method. To achieve maximum dry density, they can be compacted either in a dry state or in a saturated state by flooding with water.



For cohesionless soils, it is usual to specify a magnitude of **relative density (I_D)** that must be achieved. If ***e*** is the current void ratio or γ_d is the current dry density, the relative density is usually defined in percentage as

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \; x \; 100$$

or

$$I_D = \frac{\gamma_{dmax}(\gamma_d - \gamma_{dmin})}{\gamma_d(\gamma_{dmax} - \gamma_{dmin})} \; x \; 100$$

where e_{max} and e_{min} are the maximum and minimum void ratios that can be determined from standard tests in the laboratory, and γ_{dmin} and γ_{dmax} are the respective minimum and maximum dry densities

On the basis of relative density, sands and gravels can be grouped into different categories:

Relative Density (%)	Classification
< 15	Very Loose
15 – 35	Loose
35 – 65	Medium
65 – 85	Dense
>85	Very Dense

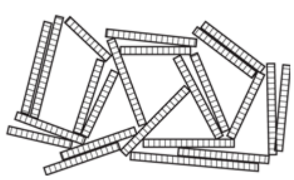
It is not possible to determine the dry density from the value of the relative density. The reason is that the values of the maximum and minimum dry densities (or void ratios) depend on the gradation and angularity of the soil grains.

Engineering Behaviour of Compacted Soils

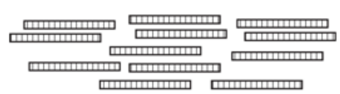
The water content of a compacted soil is expressed with reference to the OMC. Thus, soils are said to be compacted **dry of optimum** or **wet of optimum** (i.e. on **the dry side** or **wet side** of OMC). The structure of a compacted soil is not similar on both sides even when the dry density is the same, and this difference has a strong influence on the engineering characteristics.

Soil Structure

For a given compactive effort, soils have a flocculated structure on the dry side (i.e. soil particles are oriented randomly), whereas they have a dispersed structure on the wet side (i.e. particles are more oriented in a parallel arrangement perpendicular to the direction of applied stress). This is due to the well-developed adsorbed water layer (water film) surrounding each particle on the wet side.



Flocculated clay structure (Dry Side)



Dispersed clay structure (Wet Side)

Swelling

A soil on the dry side of optimum has a higher water deficiency and partially developed water films. It can therefore, imbibe more water than a soil on the wet of optimum and in the process swells more.

Shrinkage

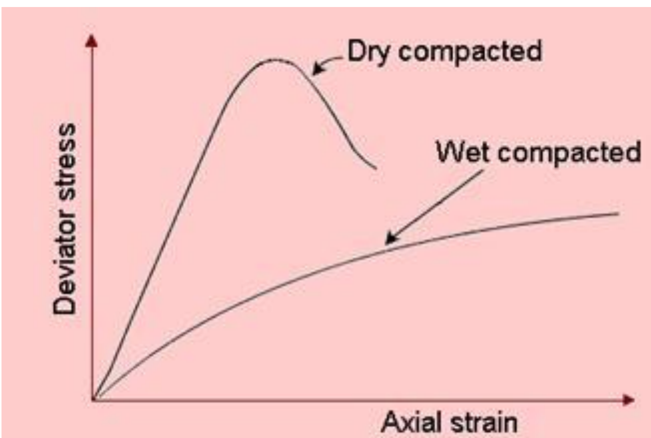
During drying, soils compacted in the wet side tend to show more shrinkage than those compacted in the dry side. In the wet side, the more orderly, nearly parallel orientation of particles allows them to pack more efficiently.

Construction pore water pressure

The compaction of man-made deposits proceeds layer by layer, and pore water pressures are induced in the previous layers. Soils compacted wet of optimum will have higher pore water pressures compared to soils compacted dry of optimum, which have initially negative pore water pressure.

Permeability
For a given compactive effort, the permeability decreases sharply with increase in water content on the dry side of optimum. The minimum permeability occurs at or slightly above the OMC. The randomly oriented soil in the dry side exhibits the same permeability in all directions, whereas the dispersed soil in the wet side is more permeable along particle orientation than across particle orientation.

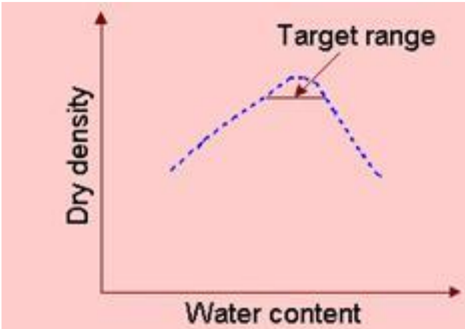
Compressibility
At low applied stresses, the dry compacted soil is less compressible on account of its truss-like arrangement of particles whereas the wet compacted soil is more compressible.



The stress-strain curve of the dry compacted soil rises to a peak and drops down when the flocculated structure collapses. At high applied stresses, the initially flocculated and the initially dispersed soil samples will have similar structures, and they exhibit similar compressibility and strength.

Field Compaction and Specifications

To control soil properties in the field during earthwork construction, it is usual to specify the **degree of compaction**(also known as the **relative compaction**). This specification is usually that a certain percentage of the maximum dry density, as found from a laboratory test (Light or Heavy Compaction), must be achieved. For example, it could be specified that field dry densities must be greater than 95% of the maximum dry density (MDD) as determined from a laboratory test. Target values for the range of water content near the optimum moisture content (OMC) to be adopted at the site can then be decided, as shown in the figure.



For this reason, it is important to have a good control over moisture content during compaction of soil layers in the field. It is then up to the field contractor to select the thickness of each soil lift (layer of soil added) and the type of field equipment in order to achieve the specified amount of compaction. The standard of field compaction is usually controlled through either end-product specifications or method specifications.

End-Product Specifications
In end-product specifications, the required field dry density is specified as a percentage of the laboratory maximum dry density, usually 90% to 95%. The target parameters are specified based on laboratory test results.

$$\text{Relative compaction} = \frac{\text{Achieved field dry density}}{\text{Laboratory maximum dry density}}$$

The field water content working range is usually within ± 2% of the laboratory optimum moisture content.

It is necessary to control the moisture content so that it is near the chosen value. From the borrow pit, if the soil is dry, water is sprinkled and mixed thoroughly before compacting. If the soil is too wet, it is excavated in advance and dried.

In the field, compaction is done in successive horizontal layers. After each layer has been compacted, the water content and the in-situ density are determined at several random locations. These are then compared with the laboratory OMC and MDD using either of these two methods: the sand replacement method, or the core cutter method.

Method Specifications
A procedure for the site is specified giving:

- Type and weight of compaction equipment
- Maximum soil layer thickness
- Number of passes for each layer

They are useful for large projects. This requires a prior knowledge of working with the borrow soils to be used.

Field Compaction Equipment

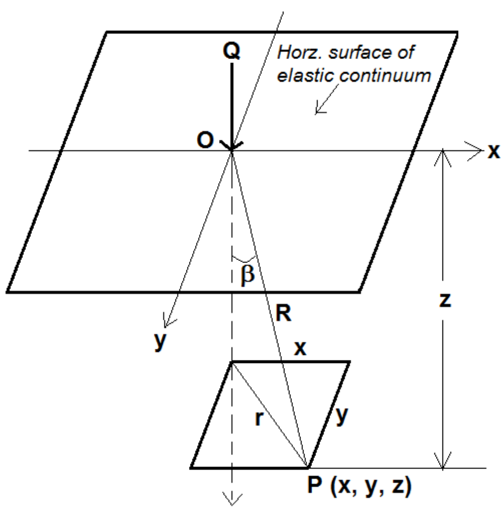
There is a wide range of compaction equipment. The compaction achieved will depend on the thickness of lift (or layer), the type of roller, the no. of passes of the roller, and the intensity of pressure on the soil. The selection of equipment depends on the soil type as indicated.

Equipment	**Most suitable soils**	**Least suitable soils**
Smooth steel drum rollers(static or vibratory)	Well-graded sand-gravel, crushed rock, asphalt	Uniform sands, silty sands, soft clays
Pneumatic tyred rollers	Most coarse and fine soils	Very soft clays
Sheepsfoot rollers	Fine grained soils, sands and gravels with > 20% fines	Uniform gravels, very coarse soils
Grid rollers	Weathered rock, well-graded coarse soils	Uniform materials, silty clays, clays
Vibrating plates	Coarse soils with 4 to 8% fines	
Tampers and rammers	All soil types	

Vertical stresses due to concentrated loads

Boussinesq gave theoretical solutions for the stress distribution in an elastic medium subjected to a concentrated load on its surface (external load). The following assumptions are made:

1. The soil mass is an elastic continuum having a constant value of modulus of elasticity (E).
2. The soil mass is homogeneous i.e. has identical properties at different points.
3. The soil is isotropic i.e. has identical properties in all directions.
4. The soil mass is semi-infinite i.e. it extends to infinity in the downward direction and lateral directions
5. The soil is weightless and free from residual stresses before the application of the load.



The polar stress σ_R at point P(x, y, z) is given by

$$\sigma_R = \frac{3}{2\pi}\frac{Q\cos\beta}{R^2}$$

where

R = polar distance between the origin O and point P

β = angle which line OP makes with the vertical

$R = \sqrt{x^2 + y^2 + z^2}$

or

$R = \sqrt{r^2 + z^2}$ where $r^2 = x^2 + y^2$

and $\sin\beta = \frac{r}{R}$ and $\cos\beta = \frac{z}{R}$

The vertical stress σ_z at point P is given by

$$\sigma_z = \sigma_R \cos^2\beta$$

$$\sigma_z = \frac{3}{2\pi}\frac{Q\cos\beta}{R^2}.\cos^2\beta$$

$$\sigma_z = \frac{3Q}{2\pi}\frac{\cos^3\beta}{R^2}$$

$$\sigma_z = \frac{3Q}{2\pi}\frac{\left(z/_R\right)^2}{R^2} = \frac{3Q}{2\pi}.\frac{z^3}{R^5}$$

$$\sigma_z = \frac{3Q}{2\pi}.\frac{1}{z^2}.\frac{z^3}{R^5}$$

$$\sigma_z = \frac{3Q}{2\pi}.\frac{1}{z^2}.\left[\frac{z^5}{(r^2+z^2)^{5/2}}\right]$$

$$\sigma_z = \frac{3Q}{2\pi}.\frac{1}{z^2}.\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$\sigma_z = I_B.\frac{Q}{z^2}$$

Where

$$I_B = \frac{3}{2\pi\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

I_B is known as Boussinesq influence coefficient for the vertical stress.

Points worth noting:

1. σ_z does not depend on E and μ. But the solution is derived assuming the soil is linearly elastic.
2. The intensity of stress just below the point load (r = 0) is

$$\sigma_z = 0.4775\frac{Q}{z^2}$$

3. At the surface s, z = 0, vertical stress just below the load is theoretically infinite.
4. The vertical stress σ_z decreases rapidly with increase in (r/z) ratio. Theoretically, the vertical stress would be zero at infinite distance from the load point. At (r/z≥5), vertical stress become extremely small and is neglected.
5. Bousssinesq solution can be applied conservatively to field problems concerning loads at shallow depths, provided the distance z is measured from the point of application of load.
6. Boussinesq solution can be applied for negative or upward loads. (Ex. Negative load is weight of soil removed.)

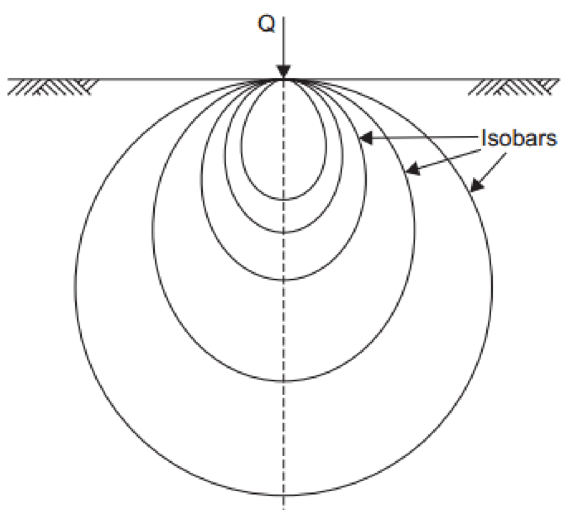
STRESS DISTRIBUTION ON A HORIZONTAL PLANE

It is possible to calculate the following pressure distributions by equation Boussinesq and present them graphically.

i. Vertical stress isobar diagram
ii. Vertical stress distribution on a horizontal plane, at a depth z below the ground surface.
iii. Vertical stress distribution along a vertical line, at a distance r from the line of action of the single concentrated load.

Isobar Diagram

An isobar is a stress contour. It is the line joining all points of equal vertical stress below the ground surface. For a particular load system, many isobars can be drawn for different chosen values of stresses. The smaller the magnitude of the selected stress, the greater the depth up to which an isobar extends. Since the vertical stress on a given horizontal plane is the same in all directions at points located at equal radial distances from the axis of loading, an isobar is a spatial curved surface of the shape of a bulb or onion.



Pressure bulb

The zone in a loaded soil mass bounded by an isobar of a given vertical pressure intensity is called a pressure bulb.

Vertical Stress Distribution on a Horizontal Plane

The vertical stress at various points on a horizontal plane at a particular depth z can be obtained using

$$\sigma_z = I_B . \frac{Q}{z^2}$$

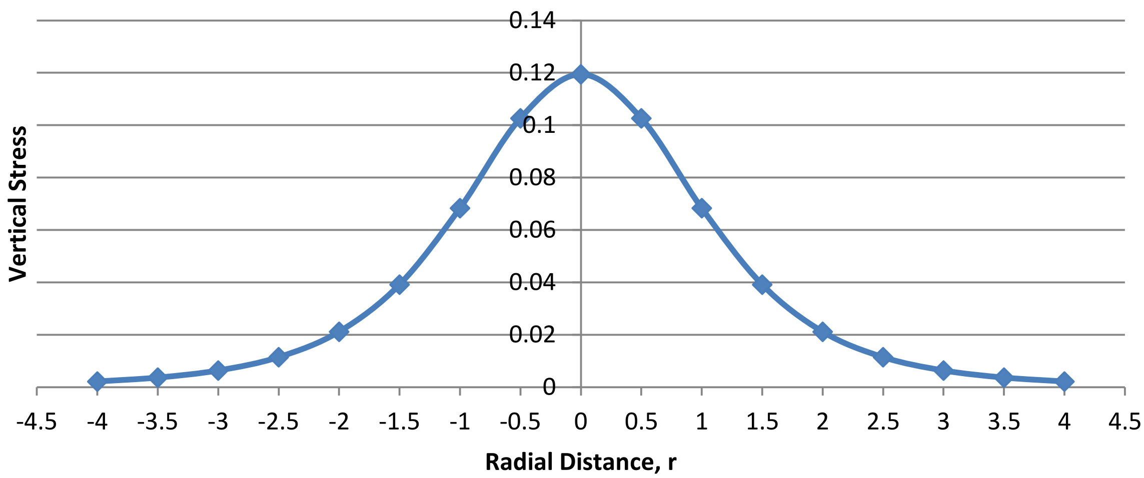
$$I_B = \frac{3}{2\pi \left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}}$$

Take z = 2 m (say), for various values of r, find r/z, I_B and σ_z.

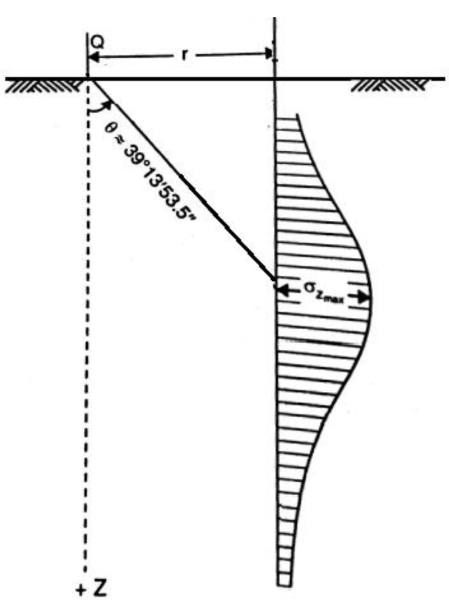
r	0	0.5	1	1.5	2	2.5	3	3.5	4
r/z	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
I_B	0.4775	0.4103	0.2733	0.1565	0.0844	0.0454	0.0251	0.0144	0.0085
σ_z	0.1194	0.1026	0.0683	0.0391	0.0211	0.0114	0.0063	0.0036	0.0021

r	**Sigma z**
-4	0.0021
-3.5	0.0036
-3	0.0063
-2.5	0.0114
-2	0.0211
-1.5	0.0391
-1	0.0683
-0.5	0.1026
0	0.1194
0.5	0.1026
1	0.0683
1.5	0.0391

2	0.0211
2.5	0.0114
3	0.0063
3.5	0.0036
4	0.0021



STRESS DISTRIBUTION ON A VERTICAL PLANE



The vertical stress distribution on a vertical line at distance r from the axis of loading: The vertical stress first increases, attains a maximum value, and then decreases. The maximum value of σ_z on a vertical line is obtained at the point of intersection of the vertical plane with a radial line at $\beta = 39^o$ 14’ through the point load. The corresponding value of r/z = 0.817.

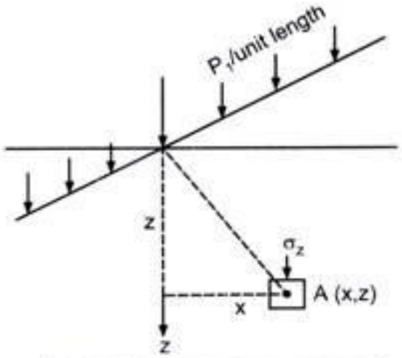
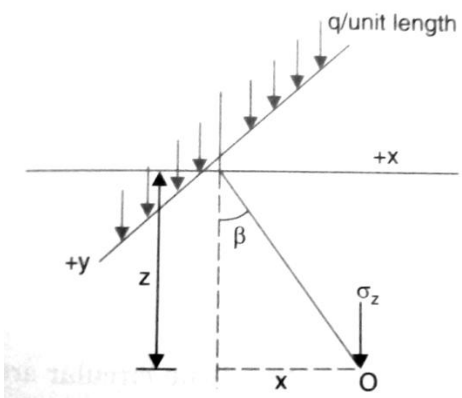


FIG. 9.6 Vertical stress due to line land.



q/unit length

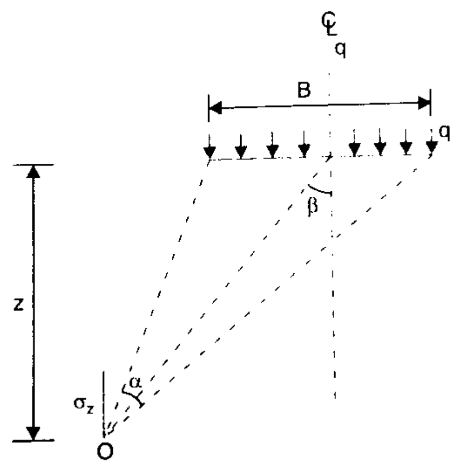


Fig. 8.4 Vertical stress due to strip load

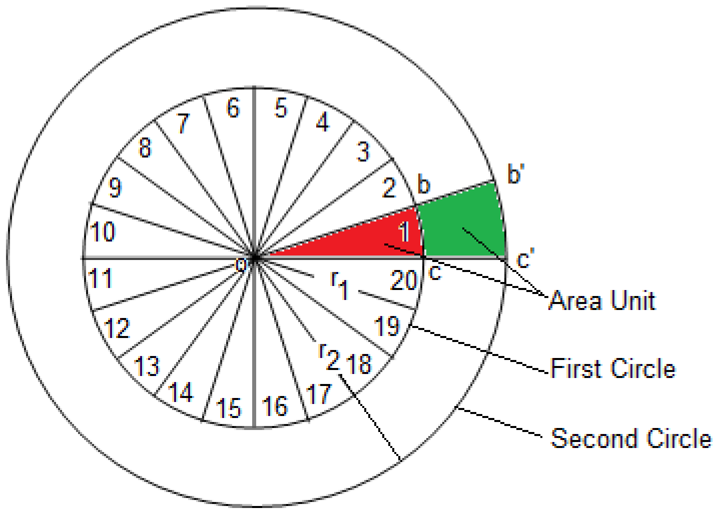
NEWMARKS INFLUENCE CHART

Based on the equation for vertical stress σ_z underneath the center of a circular loaded area with a uniformly distributed load q, Newmark developed influence charts to compute the vertical stress (and also the horizontal and shear stresses) due a loaded area of any shape, irregular or geometric, below any point either inside or outside the loaded area.

A chart consisting of number of circles and radiating lines is so prepared that the influence of each area unit, formed in the shape of a sector between two concentric circles and two adjacent radial lines, is the same at the center of the circles i.e. causes equal vertical stress at the center of the diagram.

Construction of Newmark's chart

Let a uniformly loaded circular area of radius r_1 be divided into 20 sectors. If q is the intensity of loading and σ_z the vertical stress at depth z below the center of the circular area, each area unit such as *obc* produces a stress equal to $\frac{\sigma_z}{20}$ at the point under consideration.



$$\frac{\sigma_z}{20} = \frac{q}{20}\left[1 - \left(\frac{1}{1 + \left(\frac{r_1}{z}\right)^2}\right)^{3/2}\right]$$

Let RHS of the above equation be equated to an arbitrarily fixed value, say 0.005 q. Thus,

$$\frac{q}{20}\left[1 - \left(\frac{1}{1 + \left(\frac{r_1}{z}\right)^2}\right)^{3/2}\right] = 0.005q$$

Solving the above equation,

$$\frac{r_1}{z} = 0.270$$

Thus, if a circle is drawn with radius r_1 = 0.270z and the area divided into 20 area units, each area unit will produce a vertical stress equal to 0.005q at a depth z below the centre. The arbitrarily fixed fraction 0.005 is called the influence factor.

Let a second concentric circle of radius r_2 be drawn and divided into 20 area units by extending the various radii of the first circle. Each area unit such as bb'c'c of the second circle is bounded by two radii and two arcs. Let each area unit of the second circle also produce a vertical stress of 0.005 q at depth z below the centre. Thus the total stress due to area units *obc* and *bb'c'c* at a depth z below the centre is 2 x 0.005 q.

Vertical stress due to ob'c'

$$\frac{q}{20}\left[1-\left(\frac{1}{1+\left(\frac{r_2}{z}\right)^2}\right)^{3/2}\right] = 2\ x\ 0.005q$$

Solving,

$$\frac{r_2}{z} = 0.40$$

For the tenth circle,

$$\frac{q}{20}\left[1-\left(\frac{1}{1+\left(\frac{r_{10}}{z}\right)^2}\right)^{3/2}\right] = 10\ x\ 0.005q$$

Solving,

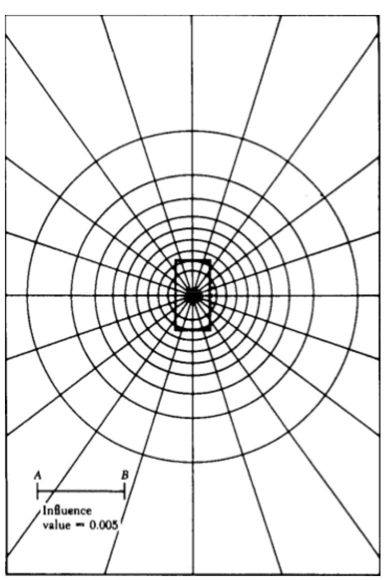
$$\frac{r_{10}}{z} = \infty$$

The r/z values for all the 10 circles are as follows:

Circle No	1	2	3	4	5	6	7	8	9	10
$\frac{r}{z}$	0.27	0.40	0.52	0.64	0.77	0.92	1.11	1.39	1.91	∞

The tenth circle lies at infinity and cannot be drawn.

Use of Newmark's Chart



1. Draw the footing shape to a scale using Length AB = Depth z on a tracing paper.

2. The point under which σ_z is required is placed at the center of the chart.

3. The number of subareas (N) including partial subareas covered by the loaded area is counted.

4. The vertical stress σ_z is computed as

$$\sigma_z = I_N . N . q$$

Where I_N is the influence factor (=0.005), N is the number of subareas covered by the loaded area, and q is the intensity of the load.