

CONSOLIDATION OF SOILS

*The property of the soil due to which a decrease in volume occurs under compressive forces is known as **compressibility of soil**.*

Compressibility is related to the magnitude of effective stress acting on the soil at that time. The compression is caused by

- Deformation and relocation of soil particles
- Compression and expulsion of air in the voids
- Expulsion of water from voids

When the soil is fully saturated, compression of soil occurs mainly due to the expulsion of water.

The compression of a saturated soil under a steady static pressure is known as consolidation.

Compression of the soil causes settlement of the structure.

Settlement is the vertical downward movement of the structure due to volume decrease of the soil on which it is built.

In general, the total settlement in soil caused by loads may be divided into three broad categories:

Immediate settlement (or elastic settlement) (S_i)

Caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content.

Immediate settlement calculations - theory of elasticity.

Primary Consolidation Settlement (S_p)

Result of a volume change in saturated cohesive soils because of expulsion of the water that occupies the void spaces.

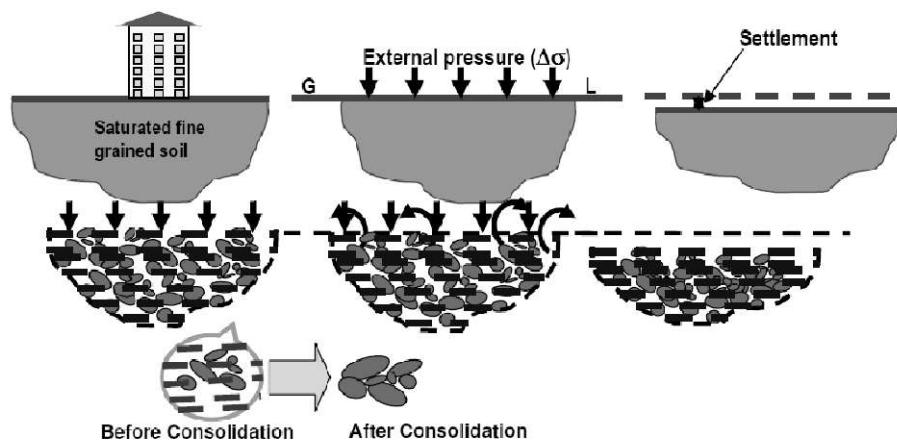
Secondary Consolidation Settlement (S_c)

Observed in saturated cohesive soils and is the result of the plastic adjustment of soil fabrics.

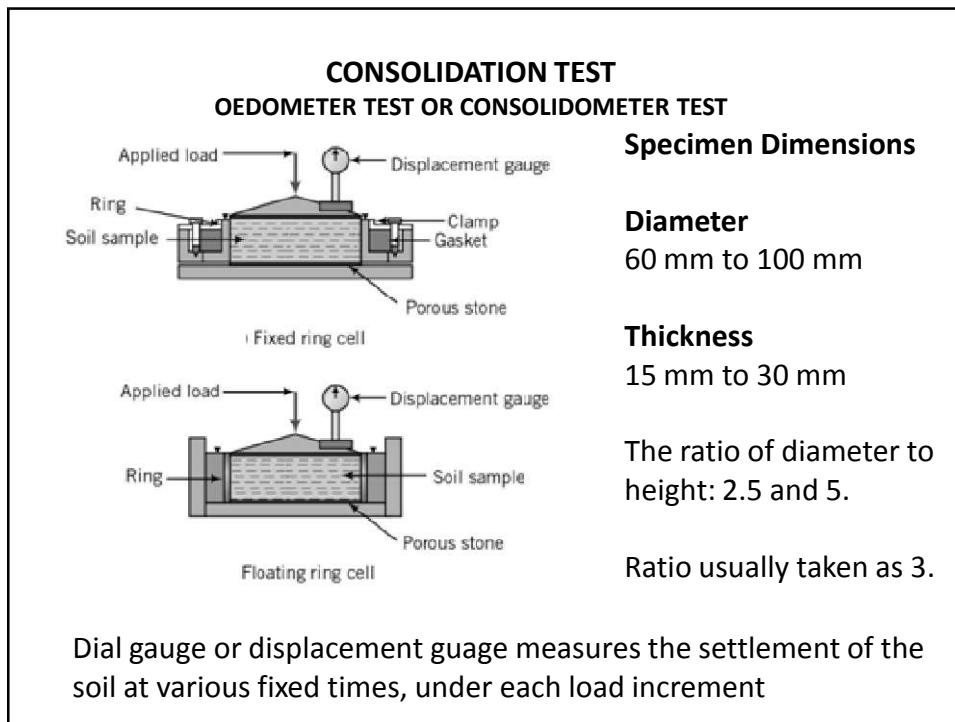
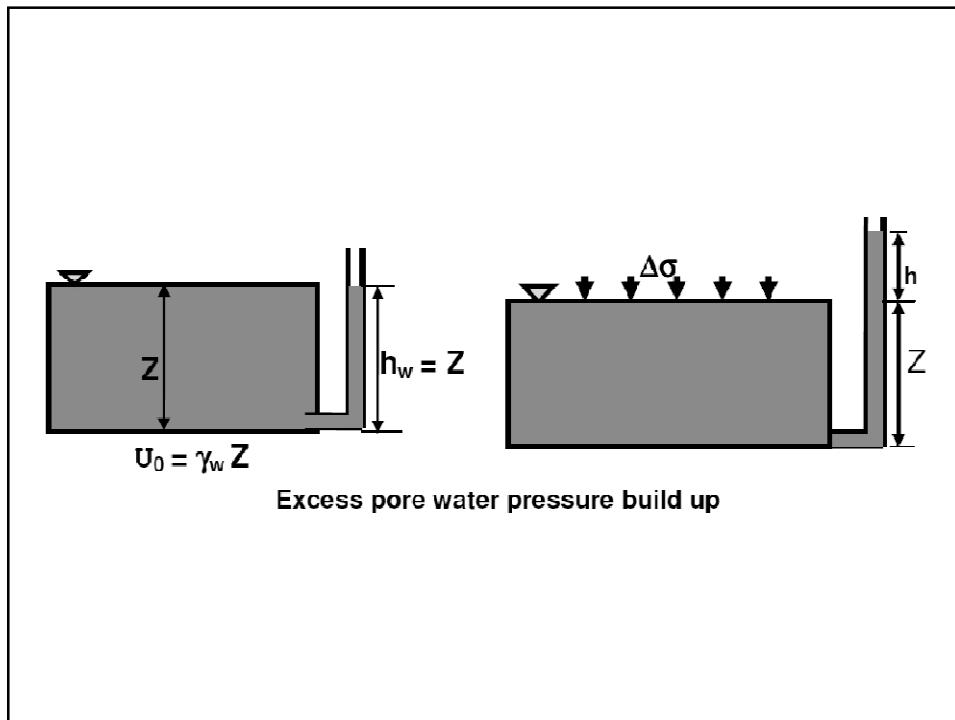
It is an additional form of compression that occurs at constant effective stress.

$$\text{Total Settlement } S = S_i + S_p + S_c$$

Compressibility of saturated soil



Mechanism of volume change in saturated fine grained soil under external loading



Initial seating load applied to prevent swelling of sample

5 kN/m² (2.5 kN/m² for very soft soils)

Duration of Loading

Till no change in dial guage reading or 24 hours, whichever is earlier.

Load Increments

Double the previous load

20, 40, 80, 160, 320 and 640 kN/m²

Or

25, 50, 100, 200, 400 and 800 kN/m²

Soil loaded until the change in settlement is negligible and the excess pore water pressure developed under current load increment has dissipated (usually 24 hours)

Longer monitoring times for exceptional soil types - montmorillonite.

For each load increment or decrement, the dial guage readings are noted at

0.25, 1.0, 2.25, 4.0, 6.25, 9.0, 12.25, 16.0, 20.25, 25, 36, 49, 64, 81, 100, 12, 144, 169, 196, 225, 289, 324, 400, 500, 600 and 1440 minutes.

After consolidation under the final load increment is complete, the loads are reduced to one-fourth of the previous load and dial guage readings are noted for each load decrement till the load is reduced to initial seating load.

Immediately after complete unloading, the ring with the sample is taken out and excess water on the sample surface is dried using a blotting paper.

The weight of ring and sample is taken. The dry mass and water content of the sample are determined.

The data obtained from the one-dimensional consolidation test

- Initial height of the soil, H_o , which is fixed by the height of the ring.
- The current height of the soil at various time intervals under each load (time-settlement data).
- The water content at the beginning and at the end of the test, and the dry weight of the soil at the end of the test.

The results of the consolidation test are plotted in the form of a plot between the void ratio and the effective stress.

Methods to determine the void ratio at various load increments:

Height of solids method – Applicable to both saturated and unsaturated soils

Change in void ratio method – Applicable only to saturated soils.

DETERMINATION OF VOID RATIO AT VARIOUS LOAD INCREMENTS

Height of solids method

The equivalent height of solids is determined from the dry mass of the soil. The height of solids is given by

$$H_s = \frac{V_s}{A} = \left(\frac{M_s}{G \rho_w} \right) \cdot \frac{1}{A}$$

Where H_s = height of solids, V_s = volume of solids, M_s = dry mass of solids, G = specific gravity of solids, A = cross sectional area of specimen.

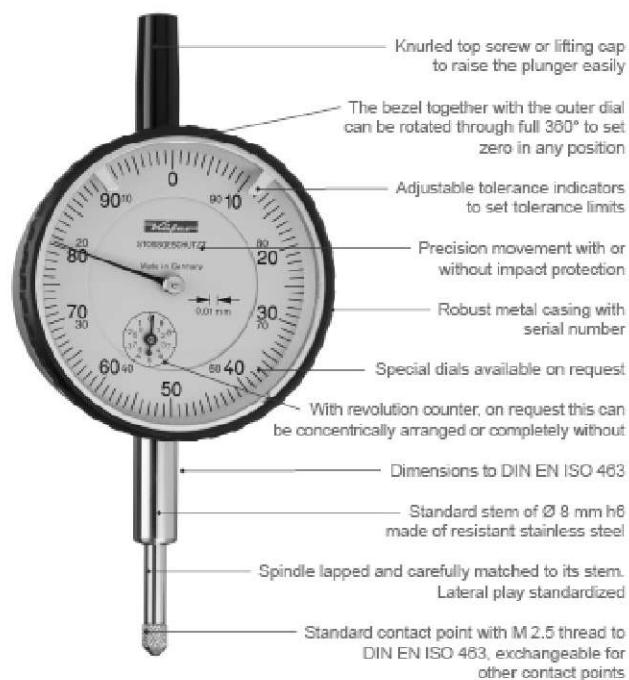
From the definition of voids ratio

$$e = \frac{V - V_s}{V_s}$$

$$e = \frac{(Ax H) - (Ax H_s)}{(Ax H_s)} = \frac{H - H_s}{H_s}$$

Where H is the total height (total thickness).

The total thickness is measured once either at the beginning or at the end of the test. At other stages of loading, the thickness H is worked out from the measured thickness and the difference in dial gauge readings.



Given data	$H_0 = 25 \text{ mm}$,	$A = 50 \text{ cm}^2$,	Volume = 125 ml,		
	$M_s = 190.24 \text{ gm}$,	$G = 2.67$,	$w_f = 24.94\%$		
Least count of dial gauge = 0.01 mm					

Observations			Calculations		
Applied pressure (kN/m ²)	Dial gauge reading	Change in thickness ΔH (mm)	$H = H_0 \pm \Sigma \Delta H$	$H - H_s$	e from Eq. (a) $= (H - H_s)/H_s$
0.0	490		25.00	10.75	0.754
10.0	482	- 0.08	24.92	10.67	0.748
20.0	470	- 0.12	24.80	10.55	0.740
40.0	431	- 0.39	24.41	10.16	0.713
80.0	390	- 0.41	24.00	9.75	0.684
160.0	343	- 0.47	23.53	9.28	0.651
320.0	295	- 0.48	23.05	8.80	0.617
640.0	249	- 0.46	22.59	8.34	0.585
0.0	364	+ 1.15	23.74	9.49	0.666

DETERMINATION OF VOID RATIO AT VARIOUS LOAD INCREMENTS

Change in void ratio method

The final void ratio e_f , corresponding to the complete swelling conditions after the load has been removed, is determined from its water content, using the equation

$$c_l = wG$$

From the definition of void ratio

$$e = \frac{V - V_s}{V_s} = \frac{V}{V_s} - 1$$

$$V = V_s(1 + e)$$

By partial differentiation,

From (a) and (b)

$$\frac{dH}{H} = \frac{de}{1+e}$$

$$\Delta e = \frac{(1 + e)\Delta II}{H}$$

ΔH is the change in thickness at the end of the test.

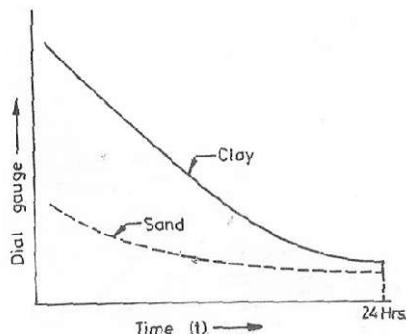
Table 12.2. Computation of Void Ratios by Change in Void Ratio Method

Given Data $H_0 = 25 \text{ mm}$, $A = 50 \text{ cm}^2$, Volume = 125 ml , $M_s = 190.24 \text{ gm}$, $G = 2.67$, $w_l = 24.94\%$, $H_l = 23.74$, $e_l = w_l \times G = 0.2494 \times 2.67 = 0.666$.
 From Eq. 12.11 (a), $\Delta e = \frac{(1+0.666)}{23.74} \times \Delta H = 0.0702 \Delta H$... (d)

Least count of dial gauge = 0.01 mm.

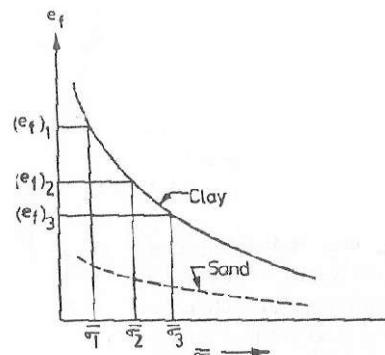
Observations			Calculations		
Applied pressure (kN/m^2)	Dial gauge reading	Change in thickness $\Delta H \text{ (mm)}$	$H \approx H_0 \pm \sum \Delta H$	Δe (from Eq. (d))	e
0.0	490	-0.08	25.00	-	0.754
10.0	482	-	24.92	- 0.006	0.748
20.0	470	- 0.12	24.80	- 0.008	0.740
40.0	431	- 0.39	24.41	- 0.027	0.713
80.0	390	- 0.41	24.00	- 0.029	0.684
160.0	343	- 0.47	23.53	- 0.033	0.651
320.0	295	- 0.48	23.05	- 0.034	0.617
640.0	249	- 0.46	22.59	- 0.032	0.585
0.0	364	+ 1.15	23.74	+ 0.081	0.666

CONSOLIDATION TEST RESULTS



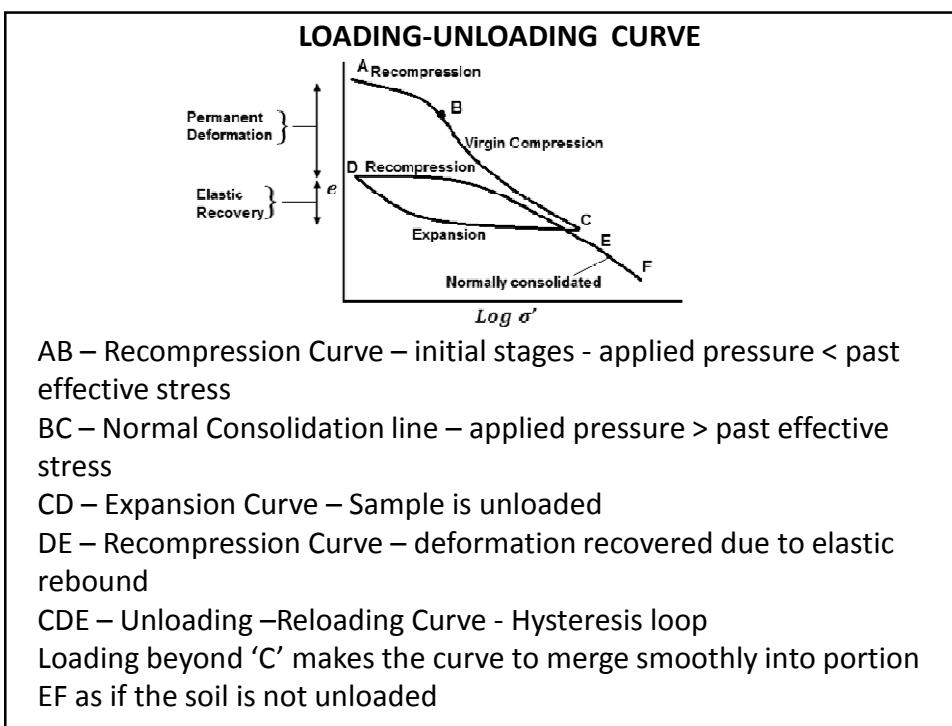
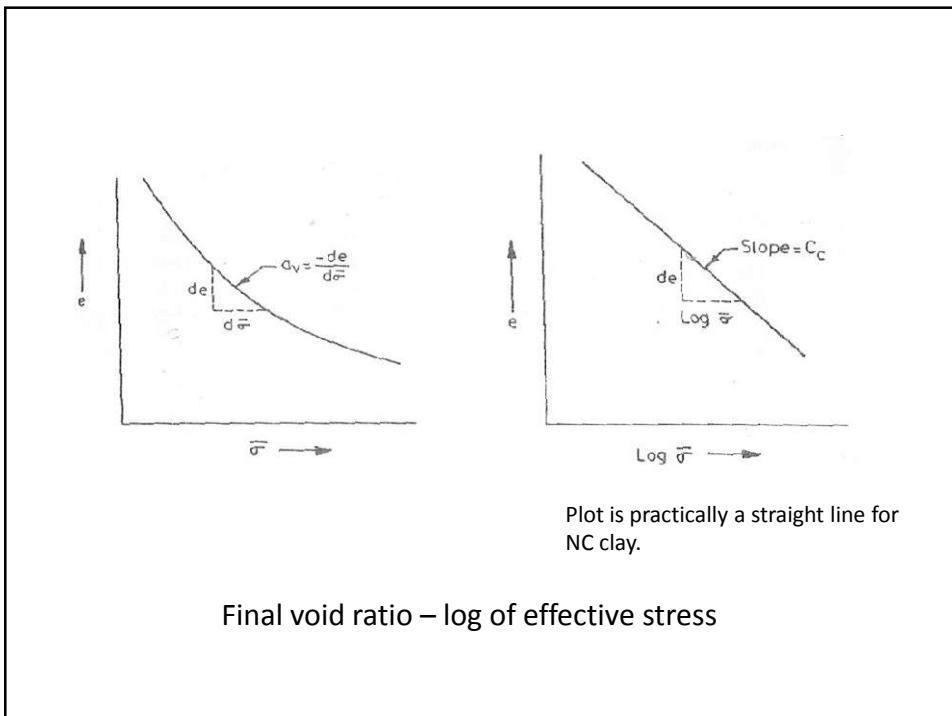
Dial gauge reading-time plot for one load increment

useful for the determination of coefficient of consolidation – in turn useful rate of consolidation in the field



Final void ratio – effective stress plot for different load increments and corresponding effective stresses

required for determination of the magnitude of the consolidation settlement in the field



PRIMARY CONSOLIDATION PARAMETERS

COEFFICIENT OF COMPRESSIBILITY, a_v

MODULUS OF VOLUME COMPRESSIBILITY, m_v

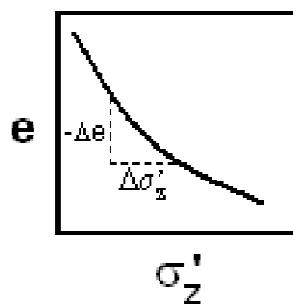
COEFFICIENT OF COMPRESSION OR COMPRESSION INDEX, C_c

EXPANSION INDEX, C_e

RECOMPRESSION INDEX, C_r

COEFFICIENT OF COMPRESSIBILITY, a_v

The coefficient of compressibility, a_v is defined as decrease in void ratio per unit increase in effective stress.



$$a_v = -\frac{(e)_2 - (e)_1}{(\sigma'_z)_2 - (\sigma'_z)_1} = \frac{|\Delta e|}{(\sigma'_z)_2 - (\sigma'_z)_1} \quad \left(\frac{\text{m}^2}{\text{kN}} \right)$$

MODULUS OF VOLUME COMPRESSIBILITY, m_v

m_v is expressed as volumetric strain per unit increase in effective stress (this is inverse of bulk modulus)

$$m_v = \frac{-\Delta V/V_o}{\Delta \sigma'_z}$$

V_o is the initial volume, ΔV is the change in volume and $\Delta \sigma'_z$ is the change in effective stress

It may also be expressed as

$$m_v = -\frac{(\varepsilon_z)_2 - (\varepsilon_z)_1}{(\sigma'_z)_2 - (\sigma'_z)_1} = \frac{|\Delta \varepsilon_z|}{(\sigma'_z)_2 - (\sigma'_z)_1} \left(\frac{m^2}{kN} \right)$$

$(\Delta V / V_o)$ can be expressed in terms of either

- (a) void ratio or the
or
- (b) thickness of the specimen.

(a) void ratio or the

Let Initial void ratio = e_o

Volume of solids = 1

Initial volume

$$V_o = (1 + e_o).$$

If Δe is the change in voids ratio due to change in volume ΔV , then $\Delta V = \Delta e$. Thus,

$$\frac{\Delta V}{V_o} = \frac{\Delta e}{1 + e_o} \quad \text{and therefore} \quad m_v = \frac{-\Delta e/(1 + e_o)}{\Delta \sigma'_z}$$

(b) thickness of the specimen

As the area of cross-section of the sample in the cosolidometer remains constant, the change in volume is also proportional to the change in height.

$$\Delta V = \Delta H$$

Therefore,

$$\frac{\Delta V}{V_o} = \frac{\Delta H}{H_o}$$

$$m_v = \frac{-\Delta H/H_o}{\Delta \sigma_z'} \quad \text{or} \quad \Delta H = -m_v H_o \Delta \sigma_z'$$

Relation between m_v and

$$m_v = \frac{a_v}{1 + e_o}$$

COEFFICIENT OF COMPRESSION OR COMPRESSION INDEX, C_c

$$C_c = \frac{e_2 - e_1}{\log \frac{(\sigma_z')_2}{(\sigma_z')_1}} = \frac{|\Delta e|}{\log \frac{(\sigma_z')_2}{(\sigma_z')_1}}$$

for undisturbed soils: $C_c = 0.009(w_L - 10)$

for remoulded soils: $C_c = 0.007(w_L - 10)$

C_c normally varies between 0.1 to 0.8

$$\text{EXPANSION INDEX, } C_e = \frac{\Delta e}{\log \frac{(\sigma_z')_2}{(\sigma_z')_1}}$$

Expansion index is much smaller than compression index

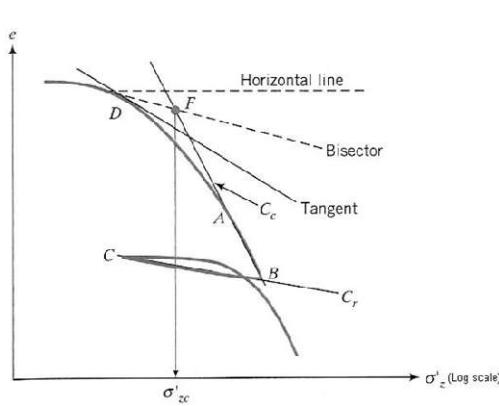
RECOMPRESSION INDEX, C_r

$$C_r = \frac{e_2 - e_1}{\log \frac{(\sigma_z')_2}{(\sigma_z')_1}} = \frac{|\Delta e|}{\log \frac{(\sigma_z')_2}{(\sigma_z')_1}}$$

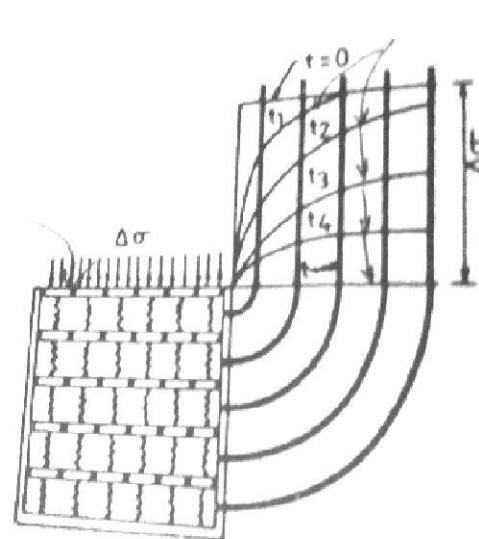
C_r is usually in the range of 1/10 to 1/5 of the compression index.

Casagrande's method to determine the pre-consolidation effective stress

Pre-consolidation effective stress is the maximum effective stress to which the soil is subjected to in the past



- Identify the point of maximum curvature, D, on the initial part of the curve.
- Draw a horizontal line through D.
- Draw a tangent to the curve at D.
- Bisect the angle formed by the tangent and horizontal line at D.
- Extend backward the straight line portion of the curve (the normal consolidation line), BA, to intersect the bisector line at F.
- The abscissa of F is the pre-consolidation stress.



Cylindrical vessel with a series of pistons separated by springs and filled with water.

Pistons are perforated to allow flow of water through the compartments

Springs represent soil skeleton – the network of soil grains

Perforation in pistons are analogous to voids that impart permeability

Immediately upon load application spring length remains unchanged since time-elapsed not sufficient to allow escape of water.

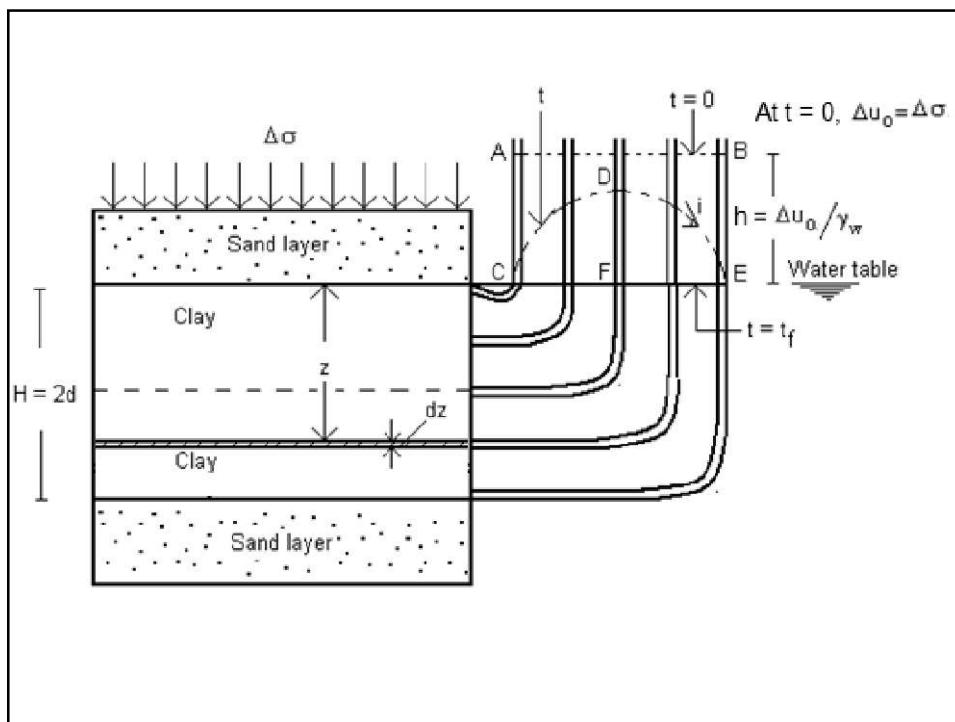
Springs cannot carry load until they compress.

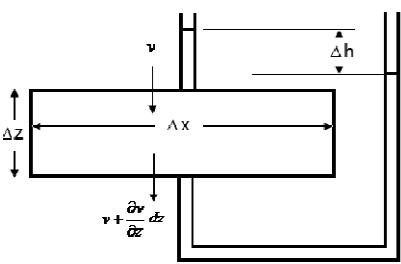
TERZAGHI'S ONE DIMENSIONAL CONSOLIDATION THEORY

(Determination of rate of consolidation)

Assumptions

1. The soil is homogeneous, isotropic and fully saturated
2. The solid particles and water in the voids are incompressible. The consolidation occurs due to expulsion of water from the voids.
3. The coefficient of permeability of the soil has the same value at all points, and its remains constant during the entire period of consolidation.
4. Darcy's law is valid throughout the consolidation process.
5. Soil is laterally confined and consolidation takes place in axial direction only. Drainage of water also occurs in vertical direction.
6. The time lag in consolidation is entirely due to low permeability of the soil.
7. Coefficient of compressibility and coefficient of volume change remain constant during the load increment





$$\Delta\sigma_z = \Delta\sigma'_z + u$$

Hydraulic gradient (i) at depth z is

$$i = \frac{\partial h}{\partial z} = \frac{\partial(u/\gamma_w)}{\partial z} = \frac{1}{\gamma_w} \left(\frac{\partial u}{\partial z} \right)$$

From Darcy's law, the velocity of flow at depth z is given by

$$v = ki = k \cdot \frac{1}{\gamma_w} \left(\frac{\partial u}{\partial z} \right)$$

The velocity of flow at the bottom of the element at depth z is given by

$$v + \frac{\partial v}{\partial z} \cdot dz = v + \frac{\partial}{\partial z} \left[\frac{k}{\gamma_w} \left(\frac{\partial u}{\partial z} \right) \right] dz \quad \text{or} \quad v + \frac{\partial v}{\partial z} \cdot dz = v + \frac{k}{\gamma_w} \left(\frac{\partial^2 u}{\partial z^2} \right) dz$$

Therefore,

$$\frac{\partial v}{\partial z} = \frac{k}{\gamma_w} \left(\frac{\partial^2 u}{\partial z^2} \right)$$

The discharge entering the element Q_{in} is

$$Q_{in} = v(\Delta x \times \Delta y)$$

Where Δx and Δy are the dimensions of the element in plan

The discharge leaving the element Q_{out} is

$$Q_{out} = \left(v + \frac{\partial v}{\partial z} \cdot dz \right) (\Delta x \times \Delta y)$$

The net discharge squeezed out of the element is given by

$$\Delta Q = \left[\left(v + \frac{\partial v}{\partial z} \cdot dz \right) - v \right] (\Delta x \times \Delta y) \quad \text{or} \quad \Delta Q = \frac{\partial v}{\partial z} (\Delta x \times \Delta y \times \Delta z)$$

As the water is squeezed out, the effective stress increases and the volume of the soil mass decreases. Therefore from the definition of modulus of volume compressibility

$$\Delta V = -m_v V_o \Delta \sigma'_z \quad \begin{aligned} V_o &= \text{initial volume of soil mass} \\ \Delta \sigma'_z &= \text{increase in effective stress} \end{aligned}$$

The decrease in volume of soil mass per unit time is

$$\frac{\partial(\Delta V)}{\partial t} = -m_v (\Delta x \times \Delta y \times \Delta z) \frac{\partial(\Delta \sigma_z^{'})}{\partial t}$$

decrease in volume of soil mass per unit time is equal to the volume of water squeezed out per unit time

$$\frac{\partial v}{\partial z} (\Delta x \times \Delta y \times \Delta z) = -m_v (\Delta x \times \Delta y \times \Delta z) \frac{\partial(\Delta \sigma_z^{'})}{\partial t}$$

$$\text{or } \frac{\partial v}{\partial z} = -m_v \frac{\partial(\Delta \sigma_z^{'})}{\partial t}$$

$$\text{Now, } \Delta \sigma_z^{'} = \Delta \sigma_z - u$$

$$\text{or } \frac{\partial(\Delta \sigma_z^{'})}{\partial t} = \frac{\partial(\Delta \sigma_z)}{\partial t} - \frac{\partial(u)}{\partial t}$$

For a given pressure increment $\partial(\Delta \sigma_z) = 0$

$$\text{Therefore, } \frac{\partial v}{\partial z} = -m_v \left(-\frac{\partial u}{\partial t} \right) = m_v \left(\frac{\partial u}{\partial t} \right)$$

From velocity at the top and bottom of the element, we have earlier shown that

$$\frac{\partial v}{\partial z} = \frac{k}{\gamma_w} \left(\frac{\partial^2 u}{\partial z^2} \right) dz$$

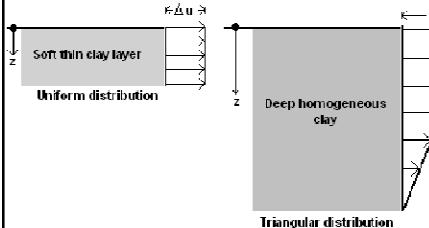
$$\text{Therefore, } \frac{k}{\gamma_w} \left(\frac{\partial^2 u}{\partial z^2} \right) dz = m_v \left(\frac{\partial u}{\partial t} \right)$$

$$\text{or } c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \quad \text{is the governing differential equation for one-dimensional consolidation. It allows to predict the changes in excess pore water pressure at various depths within the soil with time}$$

where c_v is the coefficient of consolidation given by

$$c_v = \frac{k}{m_v \gamma_w}$$

SOLUTION OF CONSOLIDATION EQUATION USING FOURIER SERIES



The boundary conditions for a uniform distribution of initial excess porewater pressure in which double drainage occurs are
 When $t = 0$, $\Delta u = \Delta u_0 = \Delta \sigma_z$.
 At the top boundary, $z = 0$, $\Delta u = 0$.
 At the bottom boundary, $z = H = 2d$, $\Delta u = 0$

A solution for the governing one-dimensional consolidation equation satisfying these boundary conditions is obtained using the Fourier series

$$u = \frac{4u_i}{\pi} \sum_{N=0}^{\infty} \frac{1}{(2N+1)} \left[\sin \frac{(2N+1)\pi z}{2d} \right] e^{-(2N+1)^2 \pi^2 T_v / 4}$$

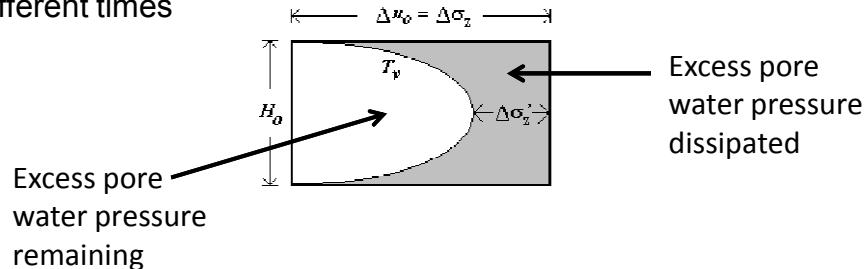
where u is the excess pore water pressure, u_i is the initial excess pore water pressure and N is a positive integer with values from 0 to ∞ , and

$$T_v = \frac{c_v t}{d^2}$$

A plot of equation

$$u = \frac{4u_i}{\pi} \sum_{N=0}^{\infty} \frac{1}{(2N+1)} \left[\sin \frac{(2N+1)\pi z}{2d} \right] e^{-(2N+1)^2 \pi^2 T_v / 4}$$

gives the variation of excess pore water pressure with depth at different times



DEGREE OF CONSOLIDATION

The degree of consolidation or consolidation ratio, U_z , which gives the amount of consolidation completed at a particular time and depth can mathematically expressed as

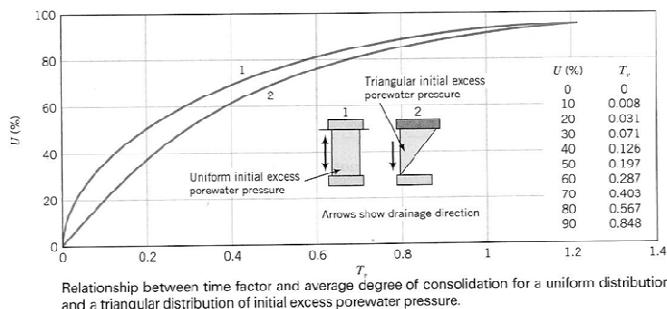
$$U_z = \frac{u_i - u}{u_i} \quad u_i - u \text{ is the dissipated excess pore water pressure}$$

where u = excess pore water pressure at some time t ,
 u_i = initial excess pore water pressure

The average degree of consolidation, U , of a whole layer at a particular time is given as

$$U = \frac{u_i - u_t}{u_i}$$

Average degree of consolidation with time factor T_v for a uniform and a triangular distribution of excess pore water pressure



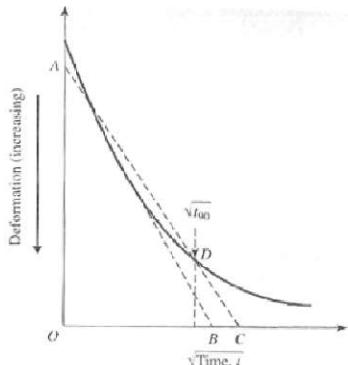
A convenient set of equations for double drainage found by curve fitting this figure is

$$T_v = \frac{\pi}{4} \left(\frac{U}{100} \right)^2 \text{ for } U < 60\%$$

$$T_v = 1.781 - 0.933 \log(100 - U\%) \text{ for } U \geq 60\%$$

U expressed as %. These equations are useful when U and T_v curves are not available.

DETERMINATION OF COEFFICIENT OF CONSOLIDATION FROM LAB OEDOMETER TEST



(a) Square root time method

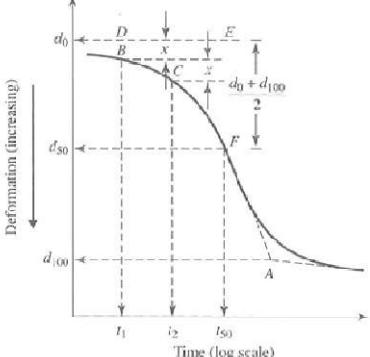
1. Plot the dial gauge readings versus square root of times.
2. Draw a line AB through the early portion of the curve.
3. Draw a line AC such that $OC = 1.15 OB$. The abscissa of point D, which is the intersection of AC and the consolidation curve, gives the square root of time for 90% consolidation ($\sqrt{t_{90}}$).
4. For 90 % consolidation, $T_v = 0.848$, therefore.

$$c_v = \frac{0.848d^2}{t_{90}}$$

where d is the length of the drainage path.

Drainage path (d): For specimens drained at both top and bottom, d equals one-half of the average height of the specimen during consolidation. For specimens drained on only one side, d equals the average height of the specimen during consolidation

(b) Logarithm of time method



1. Extend the straight line portions of the primary and secondary consolidations to intersect at point A. The ordinate of A is represented by d_{100} , the deformation at the end of 100% primary consolidation.

2. The initial curved portion of the plot of dial gauge reading versus $\log t$ is approximated to be a parabola on the natural scale. Select times t_1 and t_2 on the curved portions such that $t_2 = 4t_1$. Let the difference of specimen deformation during time $(t_2 - t_1)$ be equal to x .

3. Draw the horizontal line DE such that the vertical distance BD is equal to x . The deformation formation at 0% consolidation.

4. The ordinate of point F on the consolidation curve represents the deformation at 50% consolidation, $d_{50} = \frac{d_0 + d_{100}}{2}$, and its abscissa represents the time t_{50} .

5. For 50% average degree of the consolidation, $T_c = 0.197$, therefore

$$c_v = \frac{0.197 d^2}{t_{90}}$$

where drainage path, d , is determined in a manner similar to that in square root of time method.

RELATION BETWEEN LAB TEST AND FIELD

The time factor (T_v) provides a useful expression to estimate the settlement in the field from the results of a laboratory consolidation test. If two layers of the same clay have the same degree of consolidation, then their time factors and coefficients of consolidation are the same. Hence,

$$T_v = \frac{(C_v t)_{lab}}{(H_{at})_{lab}^2} = \frac{(C_v t)_{field}}{(H_{at})_{field}^2}$$

and, by simplification,

$$\frac{t_{field}}{t_{lab}} = \frac{(H_{at})_{field}^2}{(H_{at})_{lab}^2}$$

Primary consolidation settlement of normally consolidated fine grained soils

$$\frac{\Delta H}{H_o} = \frac{\Delta e}{1 + e_o}$$

$$\Delta H = \rho_{pc} = \frac{\Delta e}{1 + e_o} H_o$$

$$a_v = \frac{\Delta e}{\Delta \sigma'} \quad \text{or} \quad \Delta e = a_v \Delta \sigma'$$

$$\Delta H = \rho_{pc} = \left(\frac{a_v}{1 + e_o} \right) H_o \Delta \sigma'$$

$$m_v = \left(\frac{a_v}{1 + e_o} \right)$$

$$\rho_{pc} = m_v H_o \Delta \sigma'$$

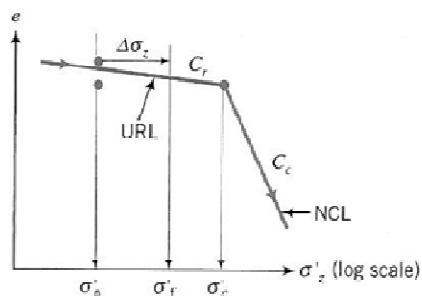
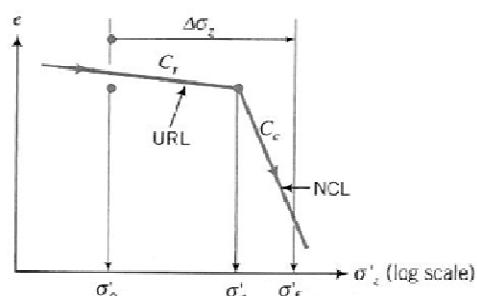
$$\frac{\Delta H}{H_o} = \frac{\Delta e}{1 + e_o}$$

$$C_c = \frac{\Delta e}{\log \frac{\sigma'_f}{\sigma'_o}} \quad \text{or} \quad \Delta e = C_c \log \frac{\sigma'_f}{\sigma'_o}$$

$$\sigma'_f = \sigma'_o + \Delta \sigma'$$

$$\Delta H = \rho_{pc} = C_c \frac{H_o}{1 + e_o} \log \frac{\sigma'_f}{\sigma'_o}$$

Primary consolidation of over-consolidated fine grained soils

(a) Case 1: $\sigma'_o + \Delta \sigma_z < \sigma'_c$ (b) Case 2: $\sigma'_o + \Delta \sigma_z > \sigma'_c$

$$\rho_{pc} = \frac{H_o}{(1 + e_o)} C_r \log \left(\frac{\sigma'_f}{\sigma'_o} \right); \sigma'_f < \sigma'_c$$

$$\rho_{pc} = \frac{H_o}{(1 + e_o)} \left(C_r \log \frac{\sigma'_f}{\sigma'_o} + C_c \log \frac{\sigma'_f}{\sigma'_c} \right); \sigma'_f > \sigma'_c$$

Procedure to calculate primary consolidation settlement

1. Calculate the current vertical effective stress (σ'_e) and the current void ratio (e_e) at the center of the soil layer for which settlement is required.
2. Calculate the applied vertical stress increase ($\Delta\sigma_z$) at the centre of the soil layer using the appropriate method
3. Calculate the final vertical effective stress, $\sigma'_f = \sigma'_e + \Delta\sigma_z$.
4. Calculate the primary consolidation settlement

- a. If the soil is normally consolidated ($OCR = 1$), the primary consolidation settlement is

$$\rho_{pe} = \frac{H_o}{(1+e_o)} C_e \log\left(\frac{\sigma'_f}{\sigma'_e}\right)$$

- b. If the soil is over consolidated and $\sigma'_f < \sigma'_{oc}$, the primary consolidation settlement is

$$\rho_{pe} = \frac{H_o}{(1+e_o)} C_e \log\left(\frac{\sigma'_f}{\sigma'_{oc}}\right)$$

- c. If the soil is over consolidated and $\sigma'_{fm} > \sigma'_{oc}$, the primary consolidation settlement is

$$\rho_{pe} = \frac{H_o}{(1+e_o)} \left(C_o \log(OCR) + C_e \log \frac{\sigma'_f}{\sigma'_e} \right)$$

where H_o is the thickness of the soil layer.