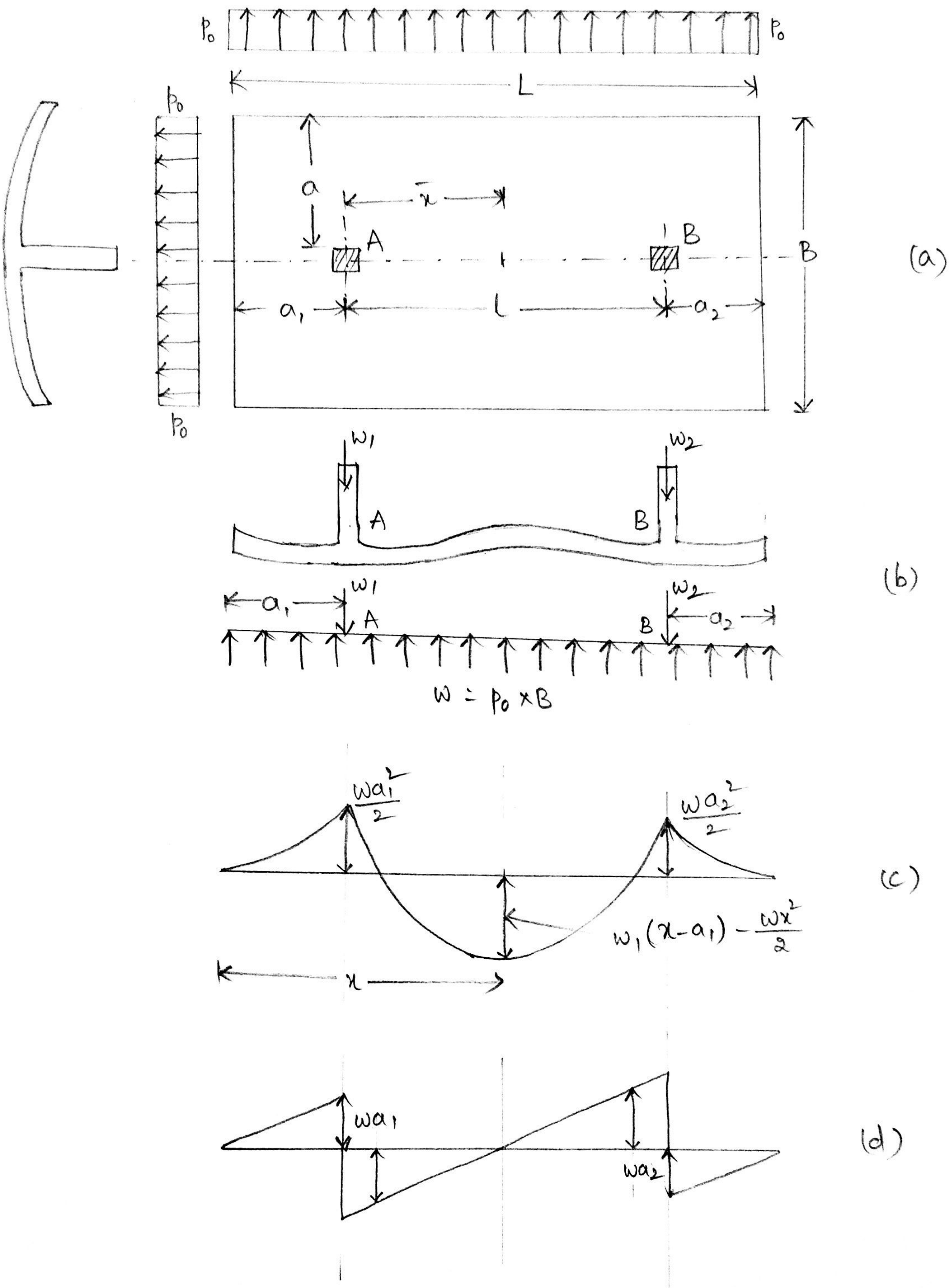
# UNIT - I

# Design of Foundations

## Combined Footings :-

* Combined footing supports the load of two (or) more adjacent columns.
* Combined footings are generally provided,
  - (i) when the columns are very near to each other so that their footings overlap
  - (ii) when the bearing capacity of soil is less, requiring more area under individual footing.
  - (iii) when the end column is near a property line, so that the footing cannot be spread in that direction.
* The combined footing may be either rectangular (or) Trapezoidal.
* The C.G of footing area should coincide with the C.G of the combined loads of two columns to get uniform pressure distribution.
* Rectangular footing is used when two columns carry equal loads.
* If one column near property line carries heavier load, Trapezoidal footing is used.
* strap and raft footings are also combined footings.

# Design of Combined Rectangular footing :-




Combined Rectangular Footing

* Fig (a) shows two columns with loads $W_1$ and $W_2$ spaced at '$l$' centre to centre.

* If $W'$ is the weight of the footing and $q_0$ is the safe bearing capacity

The footing area is given by

$$A = \frac{W_1 + W_2 + W'}{q_0}$$

Area of footing $A$ = Length × Breadth = $L \times B$

* The projections $a_1$ and $a_2$ should be so choosen that C.G of footing coincides with C.G of two loads.

$\bar{x}$ = Distance of C.G of column loads from centre of column A.

Then $a_1 + \bar{x} = L/2 = a_2 + (l - \bar{x})$

Determine $a_1$ and $a_2$.

* The net upward pressure '$p_0$' is given by

$$p_0 = \frac{W_1 + W_2}{L \times B}$$

* In the longitudinal direction, the footing will have sagging bending moment in the two cantilever portions and hogging bending moment in some length of middle portion.

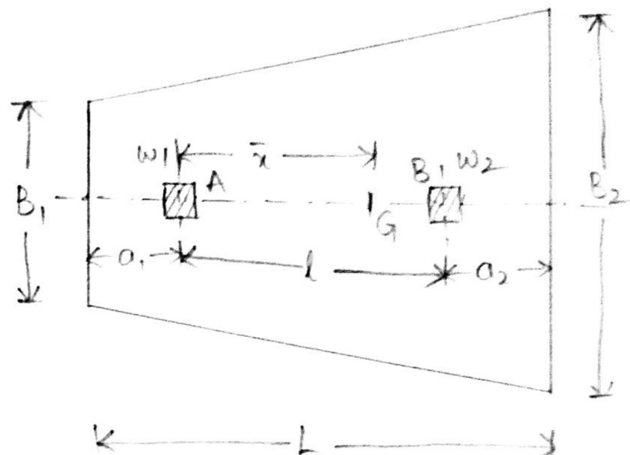
* In the transverse direction, the footing will have sagging B.M.

Explain the SFD and BMD also.

# Design of Combined Trapezoidal footing :-

* Area of footing $A = \dfrac{W_1 + W_2 + W'}{q_0}$ $\rightarrow$ ①

* The widths $B_1$ and $B_2$ are so proportioned that the C.G of footing coincides with C.G of column loads.




* Area of footing $= \dfrac{(B_1 + B_2)}{2} L$ $\rightarrow$ ②

* From ① & ② $\Rightarrow$ $B_1 + B_2 = \dfrac{2(W_1 + W_2 + W')}{q_0 L}$

* Distance $\bar{x}$ of C.G of load $= \dfrac{W_2 l}{W_1 + W_2}$

* Distance of C.G of load from short edge $= a_1 + \dfrac{W_2 l}{W_1 + W_2}$ $\rightarrow$ ③

* Distance C.G of trapezium from the short edge $= \dfrac{L}{3}\left(\dfrac{B_1 + 2B_2}{B_1 + B_2}\right)$ $\rightarrow$ ④

* Equating eqs ③ & ④

$$a_1 + \frac{W_2 l}{W_1 + W_2} = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right)$$

* the net upward pressure, $p_0 = \dfrac{W_1 + W_2}{\frac{1}{2}(B_1 + B_2) L}$

1) Design a Combined rectangular footing for two Columns A and B carrying loads of 500 kN and 700 kN respectively. Column A is 300 X 300 mm and column B is 400 X 400 mm in size. The c/c spacing of column is 3.4 m. The SBC of soil may be taken as 150 kN/m$^2$. Use $M_{20}$ and $Fe_{415}$ materials.

Sol:- Given data :-

$w_1 = 500$ kN

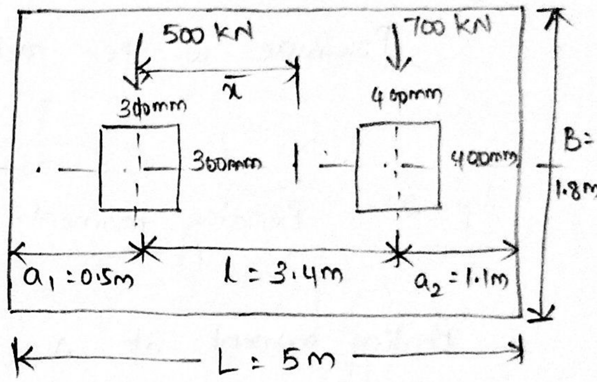
$w_2 = 700$ kN

$l = 3.4$ m

$q_o = 150$ kN/m$^2$

$f_{ck} = 20$ N/mm$^2$

$f_y = 415$ N/mm$^2$




step -1 :- Size of footing :-

Self weight of footing $w' = 10\%$ of loads from Columns A and B

$$= \frac{10}{100}(500+700) = 120 \text{ kN}$$

$\therefore$ Area of footing, $A = \dfrac{w_1 + w_2 + w'}{q_o}$

$$= \frac{500 + 700 + 120}{150} = 8.8 \text{ m}^2$$

Let the size of footing be 1.8 m × 5 m (B X L)

The distance '$\bar{x}$' of the Centre of gravity of Column loads from Centre of Column A is given by $\bar{x} = \dfrac{w_2 l}{w_1 + w_2} = \dfrac{700 \times 3.4}{500 + 700}$

$$= 1.983 \simeq 2 \text{ m}$$

We know $a_1 + \bar{x} = \frac{L}{2} = a_2 + (l - \bar{x})$

$a_1 + 2 = 5/2 \Rightarrow a_1 = 0.5\,m$

$a_2 = 2.5 - (3.4 - 2) = 1.1\,m$

$\therefore$ Net upward pressure $P_0$ is given by $P_0 = \dfrac{W_1 + W_2}{B \times L}$

$= \dfrac{500 + 700}{5 \times 1.8} = 133.33\ KN/m^2$

Pressure 'w' per metre length $= P_0 \times B = 133.33 \times 1.8$

$= 240\ KN/m$

Step-2:- Bending moment and shear forces:-

Bending moment at A

$= 240 \times \dfrac{0.5^2}{2} = 30\ KNm$

Bending moment at B

$= 240 \times \dfrac{1.1^2}{2} = 145.2\ KNm$

Maximum bending moment will occur where shear force is zero

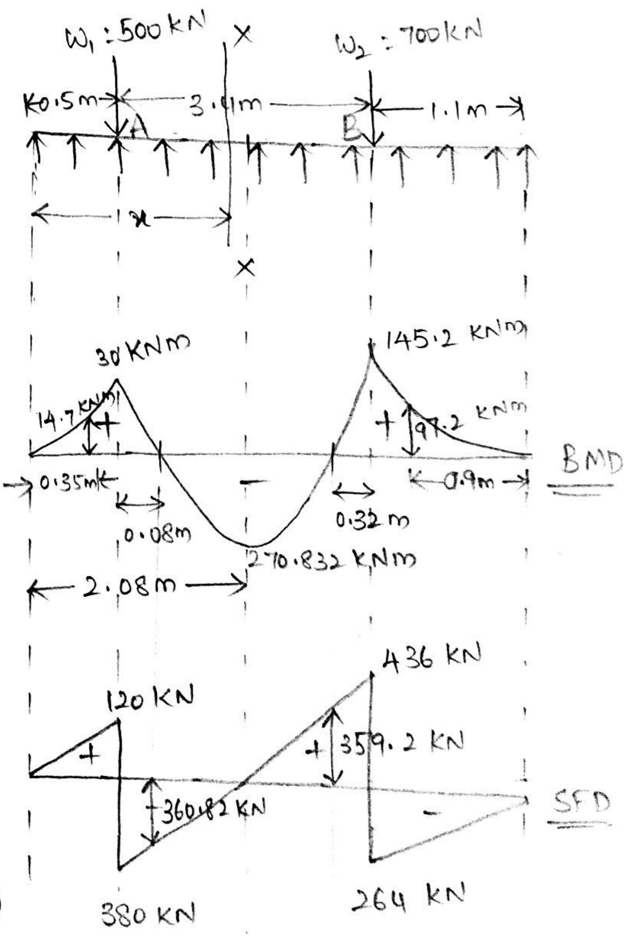
$240x - 500 = 0$

$x = 2.08\,m$ from left end

Maximum bending moment,

$M_{max} = 240 \times \dfrac{2.08^2}{2} - 500\,(2.08 - 0.5)$

$= -270.832\ KNm$



Sagging B.M at outer face of column A = $\frac{240 \times (0.5-0.15)^2}{2}$

$= 14.7$ KNm

Sagging B.M at outer face of column B = $\frac{240 \times (1.1-0.2)^2}{2} = 97.2$ KNm

Let x' be the distance of point of contraflexure from left end.

For point of contraflexures equate B.M to zero.

$$240\frac{x^2}{2} - 500(x-0.5) = 0$$

$$120x^2 - 500x + 250 = 0$$

$x = 3.58$ m and $0.58$ m from left end.

Shear force :-

shear force Just left to A = $240 \times 0.5 = 120$ kN

shear force Just right to A = $240 \times 0.5 - 500 = -380$ kN

shear force Just right to B = $-240 \times 1.1 = -264$ KN

shear force Just left to B = $-240 \times 1.1 + 700 = 436$ kN

shear force at contraflexures,

shear force at 3.58m = $240 \times 3.58 - 500$ = 359.2 ~~360.8~~ kN

shear force at 0.58m = $240 \times 0.58 - 500 = -360.8$ KN

step-3:- Effective depth :- Depth from bending compression :-

~~Provide~~ ~~an~~ Effective depth from bending compression,

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{270.832 \times 10^6}{0.138 \times 20 \times 1800}} = 286 \text{ mm}$$

233 mm

Provide an effective depth of 300 mm.

Step - 4 :- check for Punching shear :-

Let us check the above depth for punching shear, for which the critical plane lies at $d/2 = 150$ mm around the column B.

width 'bo' of critical plane = 400 + 300 = 700 mm

Punching shear force at column B = $1.5\,(W_2 - p_0 \times b_0^2)$

$= 1.5\left(700 - \frac{400}{3} \times 0.7^2\right) = 952$ kN

shear stress, $\tau_v = \frac{952 \times 10^3}{4 \times b_0 \times d} = \frac{952 \times 10^3}{4 \times 700 \times 300} = 1.133$ N/mm²

Permissible shear stress = $K_s \tau_c$

For $D \geq 300$ mm, $K_s = 1$

$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118$ N/mm²

∴ permissible shear stress = 1.118 N/mm²

$\tau_v > K_s \tau_c$

Hence the depth found from bending compression is inadequate.

The required depth, $d = \frac{V_u}{4\, b_0\, \tau_v}$

$= \frac{952 \times 10^3}{4 \times 700 \times 1.118} = 304.12 = 305$ mm

using 60 mm cover to the centre of reinforcement

overall depth D = 305 + 60 = 365 mm

Hence provide overall depth, D = 370mm and d = 370 - 60 = 310 mm step-5:-

## Design for bending tension in Longitudinal direction:-

(i) Reinforcement for Hogging bending moment :-

$$M_u = 0.87\, f_y\, A_{st}\, d \left(1 - \frac{f_y A_{st}}{f_{ck}\, b d}\right)$$

$$1.5 \times 270.83 \times 10^6 = 0.87 \times 415 \times A_{st} \times 310 \left(1 - \frac{415 A_{st}}{20 \times 1800 \times 310}\right)$$

$$406.24 \times 10^6 = 111925.5\, A_{st} - 4.162\, A_{st}^2$$

$$A_{st} = 4328.62\ mm^2$$

Using 16 mm dia bars, No. of bars, $n = \dfrac{4328.62}{\frac{\pi}{4} \times 16^2} = 21.6 \simeq 22$ no's

$\left( p_t = \dfrac{100 \times 22 \times \frac{\pi}{4} \times 16^2}{1800 \times 310} = 0.79\% > 0.12\% \right)$

Provide 22 bars of 16 mm dia uniformly distributed and Continue all the bars upto point of Contraflexure.

From the requirements of Codal provisions, we have to check the development length. (cl 26.2.3.3 (c), Pg 44)

$$\frac{M_1}{V_u} + L_o \geq L_d$$

$$\text{Actual } A_{st} = \frac{\pi}{4} \times 16^2 \times 22 = 4423.9\ mm^2$$

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b} = \frac{0.87 \times 415 \times 4423.9}{0.36 \times 20 \times 1800} = 123.24\ mm$$

$$M_1 = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$$

$$= 0.87 \times 415 \times 4423.9\, (310 - 0.42 \times 123.24) = 412.4\ kNm$$

At the point of Contraflexure near column B

shear force, $V_u = 1.5 \times 359.2 = 538.8$ KN

$L_o = d$ (or) $12 \times$ diameter — which ever is ~~less~~ greater

$= 310$ mm (or) $12 \times 16 = 192$ mm

$\therefore L_o = 310\,mm$

$L_d = 47 \times \phi = 47 \times 16 = 752\,mm$

$\therefore \dfrac{412.4 \times 10^6}{538.8 \times 10^3} + 310 = 1075\,mm > L_d = 752\,mm$

Hence Codal requirements are satisfied.

However all the 22 bars are taken upto the outer faces of both the columns. Out of these 14 bars are curtailed and remaing 8 bars are taken upto the either edge of the footing. These bars serves as a anchorage for the Stirrups.

(ii) Reinforcement for sagging bending moment :-

$M_{2u} = 97.2 \times 10^6 \times 1.5 = 145.8 \times 10^6\ Nmm$

$$145.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 310 \left(1 - \frac{415\, A_{st}}{20 \times 1800 \times 310}\right)$$

$A_{st} = 1369.73\,mm^2$

Using 12 mm dia bars, No. of bars, $n = \dfrac{1369.73}{\pi/4 \times 12^2} = 12.1 \simeq 13$ bars.

$p_t = \dfrac{100 \times 13 \times \pi/4 \times 12^2}{1800 \times 310} = 0.263\,\% > 0.12\,\%$

Provide 13 bars of 12mm diameter.

Actual $A_{st} = 13 \times \pi/4 \times 12^2 = 1470.46\,mm^2$

$$x_u = \frac{0.87 \times 415 \times 1470.46}{0.36 \times 20 \times 1800} = 40.96\,mm$$

$M_2 = 0.87 \times 415 \times 1470.46\,(310 - 0.42 \times 40.96)$

$= 155.4\ kNm$

$L_o = 310\,mm$, $V_u = 538.8\ kN$

$L_d = 47 \times 12 = 564\,mm$

$$\frac{M_2}{V_u} + L_o = \frac{155.4 \times 10^6}{538.8 \times 10^3} + 310 = 598\,mm > L_d = 564\,mm.$$

Hence Codal requirements are satisfied.

Provide 13 No's of 12mm diameter. These bars are extended by $L_o = 310mm$ beyond the point of Contraflexure. At this point curtail 5 bars and continue 8 bars through out the length to serve as anchor bars for stirrups.

(iii) Reinforcement for Sagging bending moment :-

$$M_{3u} = 1.5 \times 14.7 \times 10^6 = 22.05 \times 10^6 \text{ N mm}$$

$$22.05 \times 10^6 = 0.87 \times 415 \times A_{st} \times 310 \left(1 - \frac{415\,A_{st}}{20 \times 1800 \times 310}\right)$$

$$A_{st} = 202.55 \text{ mm}^2$$

$$A_{st\,min} = 0.12\% \text{ Gross c/s area}$$

$$= \frac{0.12}{100} \times 1800 \times 370 = 799.2 \text{ mm}^2$$

Using 12 mm dia bars, No. of bars $= \dfrac{799.2}{\pi/4 \times 12^2} = 7.06 \simeq 8$ bars

$$A_{st\,provided} = \pi/4 \times 12^2 \times 8 = 904.89 \text{ mm}^2$$

$$x_u = \frac{0.87 \times 415 \times 904.89}{0.36 \times 20 \times 1800} = 25.2 \text{ mm}$$

$$M_3 = 0.87 \times 415 \times 904.89\,(310 - 0.42 \times 25.2)$$

97.8

$$= 36.85 \text{ KNm}$$

$$V_u = 538.8 \text{ KN}$$

$$L_o = 310 \text{mm}$$

$$L_d = 47 \times 12 = 564 \text{ mm}$$

$$\frac{M_3}{V_u} + L_o = \frac{36.85 \times 10^6}{538.8 \times 10^3} + 310 = 378 \text{ mm} < L_d = 564 \text{ mm}$$

Hence not safe.

Let 'n' be the no. of bars required, $\frac{x_{umax}}{d} = 0.48$

Let us assume that $\frac{x_u}{d} = 0.3$ for under reinforced section.

$$x_u = 0.3d = 0.3 \times 310 = 93 \text{ mm}$$

$$M_3 = 0.87 \times 415 \times \pi/4 \times 12^2 \times n \ (310 - 0.42 \times 93)$$

$$= 11.064 \times 10^6 \, n \text{ N mm}$$

$$\frac{11.064 \times 10^6 \, n}{538.8 \times 10^3} + 310 = 564 \Rightarrow n = 12.4 \simeq 13 \text{ bars}$$

Provide 13 no's of 12 mm diameter. Continue these bars upto 310 mm beyond the point of Contraflexure. At this point curtail 5 bars and Continue remaining 8 bars to serve as anchor bars for stirrups.

step-6:- Check for one way shear:-

In the cantilever portion, test for one way shear is made at a distance d = 310 mm from the column face (or) 310 + 200 = 510 mm from the Centre of Column 'B'.

$$\text{shear force, } V_u = 1.5\ (264 - 240 \times 0.51) = 212.4 \text{ kN}$$

$$\text{Nominal shear stress } \tau_v = \frac{V_u}{bd}$$

$$= \frac{212.4 \times 10^3}{1800 \times 310} = 0.38 \text{ N/mm}^2$$

$$P_t = \frac{100 A_{st}}{bd} = \frac{100 \times 13 \times \pi/4 \times 12^2}{1800 \times 310} = 0.263 \%$$

Permissible shear stress = $K \tau_c$

For D = 370 mm, K = 1

For $P_t = 0.263\%$, $\tau_c = 0.34 \text{ N/mm}^2$

$\tau_v > K_s \tau_c$

Hence shear reinforcement is required.

Using 8 mm diameter 8 legged stirrups, spacing $S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$

$$S_v = \frac{0.87 \times 415 \times \pi/4 \times 8^2 \times 8 \times 310}{212.4 \times 10^3} = 211.92\ mm$$

Provide 8mm diameter 8 legged stirrups @ 200mm c/c. Provide

Similar stirrups in the other cantilever portion near column A.

Step-7:- Transverse reinforcement :-

(i) For Column A :-

The footing will bent transversely near each column face.

Projection 'a' beyond column face of A = $\frac{1}{2}(1800 - 300) = 750mm$

width $B_1$ of bending strip = $b + 2d = 300 + 2 \times 310 = 920mm$

Net upward pressure, $p_o' = \frac{W_1}{B \times B_1} = \frac{500 \times 10^3}{1800 \times 920} = 0.3019\ N/mm^2$

$= 301.9\ kN/m^2$

Maximum bending moment will occur at the face of column.

$$M = p_o' \times a^2/2 = 301.9 \times 0.75^2/2 = 84.9\ kNm$$

$$M_u = 1.5 \times 84.9 = 127.36\ kNm$$

$$d = \sqrt{\frac{M_u}{0.138 f_{ck} b}} = \sqrt{\frac{127.36 \times 10^6}{0.138 \times 20 \times 1000}} = 215\ mm$$

∴ Actual d = 310mm, provided depth is sufficient. The transverse

beam will be of same thickness.

The transverse reinforcement will be provided above the longitudinal reinforcement.

$d = 310 - 12 = 298$ mm

$$127.36 \times 10^6 = 0.87 \times 415\, A_{st} \times 298 \left(1 - \frac{415 \times A_{st}}{20 \times 1000 \times 298}\right)$$

$$= 107592.9\, A_{st} - 7.49\, A_{st}^2$$

$A_{st} = 1301.67\ mm^2$

Using 12 mm dia bars, Spacing $S = 86$ mm

Provide 12mm dia bars @ 80 mm c/c. The reinforcement provided above should have sufficient development length, $L_d = 47 \times 12 = 564$ mm.

Using 60 mm clear cover on the sides, length of bar available

$= 750 - 60 = 690$ mm $> 564$ mm.

(ii) for Column B:-

Projection $a = \frac{1}{2} \times (1800 - 400) = 700$ mm

width, $B_1 = b + 2d = 400 + 2 \times 310 = 1020$ mm

Net upward pressure, $p_0' = \frac{W_2}{B \times B_1} = \frac{700 \times 10^3}{1800 \times 1020} = 381.26\ KN/m^2$

$M_u = 1.5\, p_0'\, a^2/2 = 1.5 \times 381.26 \times 0.7^2/2 = 140.11$ KNm

$$d = \sqrt{\frac{140.11 \times 10^6}{0.138 \times 20 \times 1000}} = 225.3 = 226 \text{ mm}$$

Actual $d = 310$ mm

$d = 310 - 12 = 298$ mm for transverse reinforcement

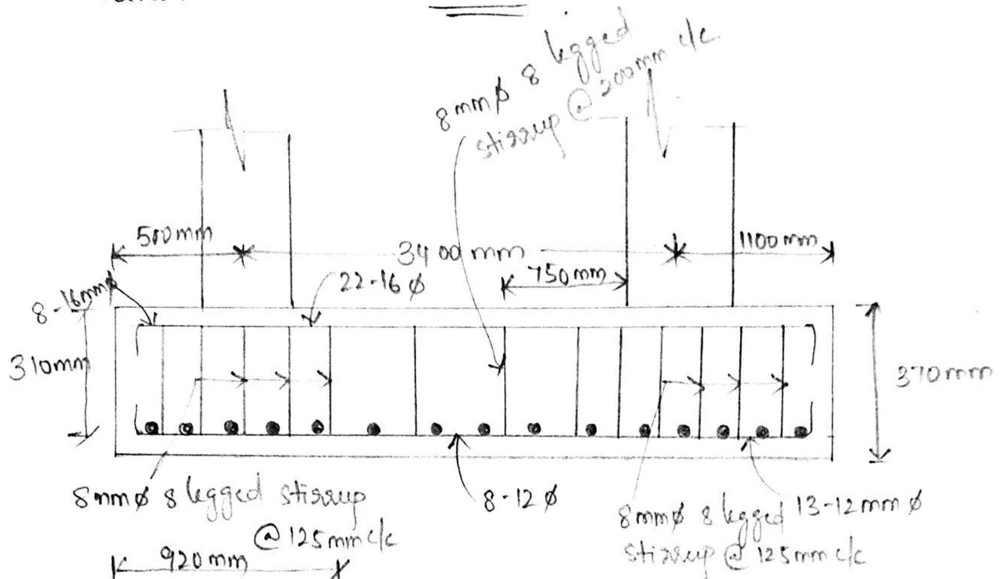
$140.11 \times 10^6 = 107592.9\, A_{st} - 7.49\, A_{st}^2$

$A_{st} = 1448\ mm^2$

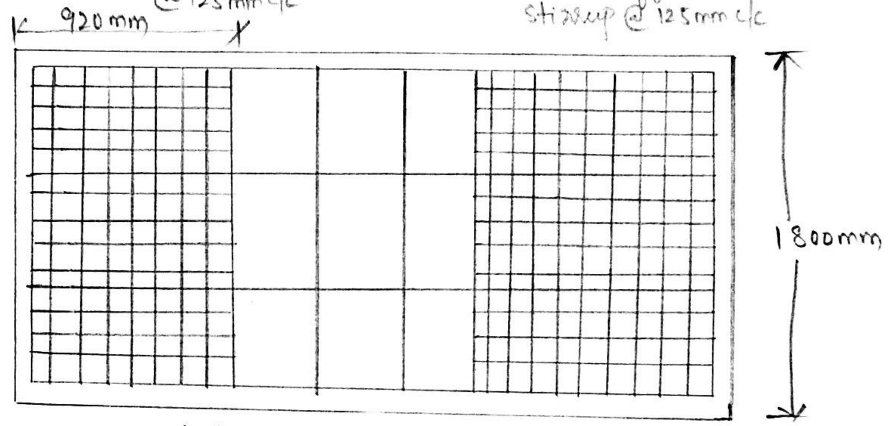
Using 12mm dia bars, spacing $S = 78$ mm

$L_d$ required $= 47 \times 12 = 564$ mm

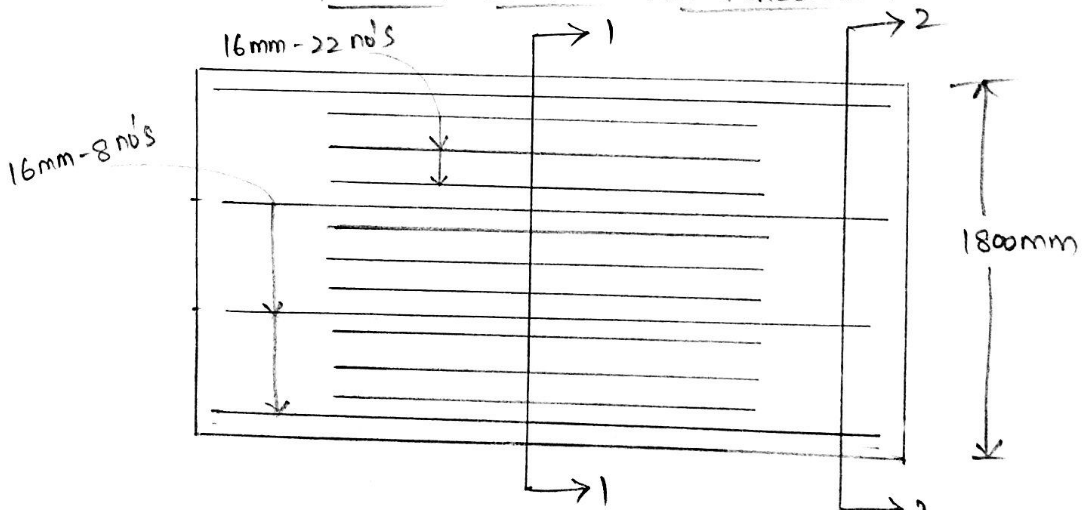
$L_d$ available $= 700 - 60 = 630$mm $> 564$ mm



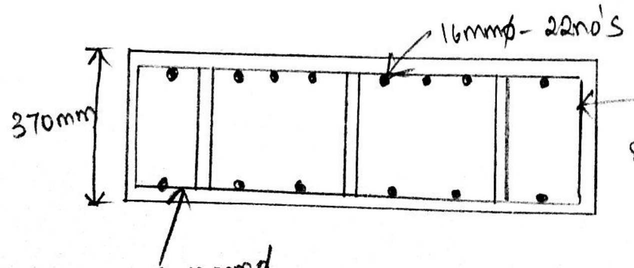

Elevation



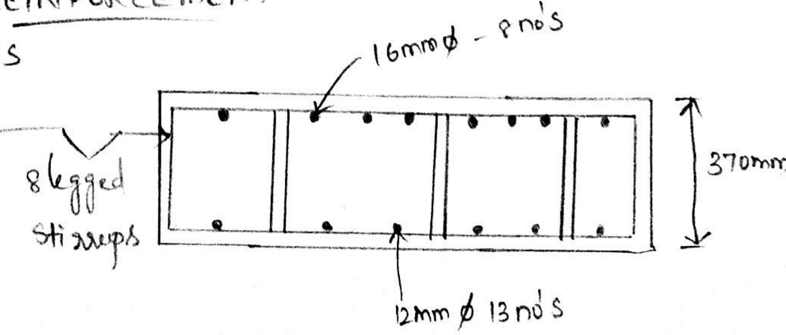

PLAN OF BOTTOM REINFORCEMENT




PLAN OF TOP REINFORCEMENT




Section 1-1




Section 2-2

2) Design a Combined trapezoidal footing for two columns A and B, Spaced 5m c/c. Column A is 300 x 300mm in size and transmits a load of 600 KN. Column B is 400 x 400mm in size and carries a load of 900 KN. The maximum length of footing is restricted to 7m only. The safe bearing capacity of soil can be taken as 120 KN/m². Use $M_{20}$ mix for Concrete and Fe 415 steel.

Sol:- Given data :-

$W_1 = 600$ KN

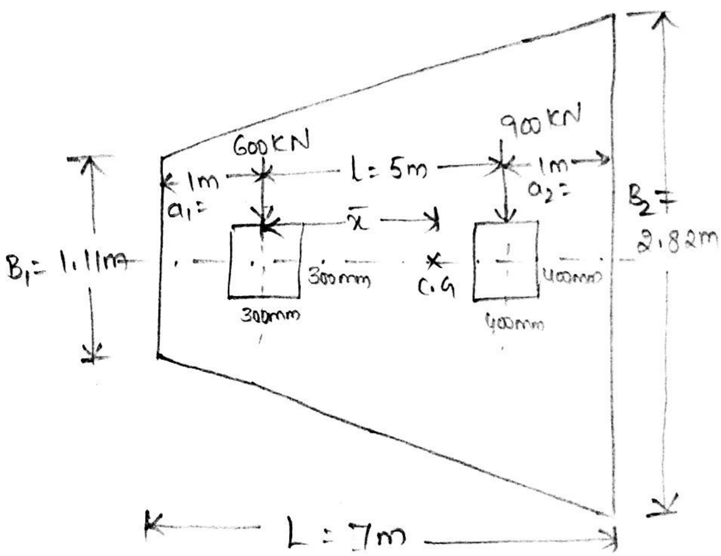
$W_2 = 900$ KN

$l = 5m$, $L = 5m$

$q_0 = 120$ KN/m²

$f_{ck} = 20$ N/mm²

$f_y = 415$ N/mm²




step-1:- Size of footing :-

self weight of footing, $w' = 10\%$ of loads from Columns A and B

$= \frac{10}{100}(600 + 900) = 150$ KN

∴ Area of footing, $A = \frac{W_1 + W_2 + W'}{q_0} = 13.75 \, m^2$

$A = \frac{1}{2}(B_1 + B_2) L = 13.75 \, m^2$

$\Rightarrow B_1 + B_2 = 3.93 \, m \longrightarrow (1)$

The distance $\bar{x}$ of the centre of gravity of column loads from centre of column A is given by $\bar{x} = \frac{W_2 \, l}{W_1 + W_2} = \frac{900 \times 5}{600 + 900}$

$= 3 \, m$

We know $a_1 + \frac{w_2 l}{w_1 + w_2} = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right)$

Let us keep $a_1 = a_2 = \frac{1}{2}(7-5) = 1m$

$$1 + \frac{900 \times 5}{600 + 900} = \frac{7}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right)$$

$$\frac{12}{7} = \frac{B_1 + 2B_2}{B_1 + B_2} \Rightarrow 5B_1 - 2B_2 = 0 \Rightarrow B_1 = \frac{2}{5} B_2 \rightarrow (2)$$

Substitute (2) in (1) $\Rightarrow \frac{2}{5} B_2 + B_2 = 3.93$

$$\Rightarrow B_2 = 2.82\,m$$

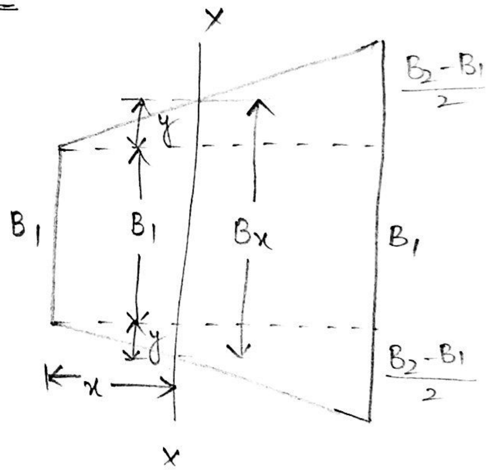
$$\therefore B_1 = 1.11m$$

Net upward pressure $p_o$ is given by $p_o = \frac{W_1 + W_2}{\frac{1}{2}(B_1 + B_2)L}$

$$= \frac{600 + 900}{\frac{1}{2}(3.93) \times 7} = 109.05 \simeq 110\ kN/m^2$$

Step-2:- Bending moment and shear forces:-

Consider any section at a distance 'x' from left end.




Let $B_x$ = width of footing at $x-x$

$$= B_1 + 2y$$

From similar triangles

$$\frac{y}{x} = \frac{(B_2 - B_1)/2}{L} = \frac{(2.82 - 1.11)/2}{7} = 0.122$$

$$y = 0.122x$$

$$\therefore B_x = B_1 + 2 \times 0.122x = 1.11 + 0.244x \rightarrow (3)$$

Area of footing at $x-x$ $= \frac{B_1 + B_x}{2} x = A_{fx}$

$$= \frac{1.11 + 1.11 + 0.244x}{2} \times x = 1.11x + 0.122x^2 \rightarrow (4)$$

The total upward force, $V_x = P_0 \times A_{fx}$

$$= 110\left(1.11x + 0.122x^2\right) \longrightarrow (5)$$

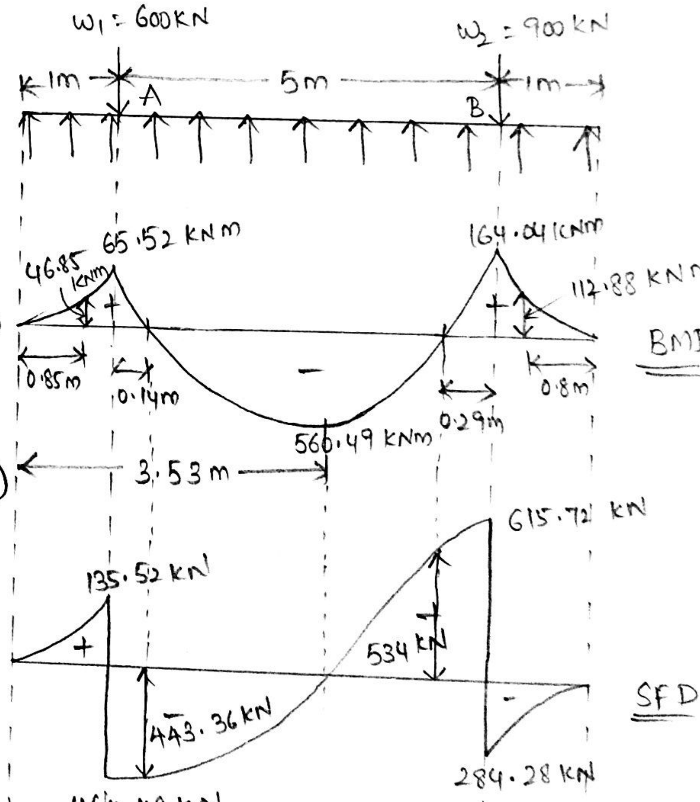
Bending moment, $M_x = \int_0^x V_x \, dx$

$$= \int_0^x 110\left(1.11x + 0.122x^2\right) dx$$

$$= 110\left(1.11\,x^2/2 + 0.122\,x^3/3\right)_0^x$$

$$M_x = \frac{110}{6}\left(3.33x^2 + 0.244x^3\right) \longrightarrow (6)$$

For calculation of shear force and bending moment, eqns (5) & (6) are used.




Bending moments :-

$$M_x = \frac{110}{6}\left(3.33x^2 + 0.244x^3\right)$$

B.M at A, $x = 1m$

$$= \frac{110}{6}\left(3.33 + 0.244\right) = 65.52 \text{ kNm}$$

B.M at B, $x = 6m$

$$= \frac{110}{6}\left(3.33 \times 6^2 + 0.244 \times 6^3\right) - 600 \times (5-1)$$

$$= 164.04 \text{ KNm}$$

Maximum bending moment occur where shear force is zero.

$$110\left(1.11x + 0.122x^2\right) - 600 = 0$$

$$x = 3.53 m$$

$$M_{max} = \frac{110}{6}\left(3.33 \times 3.53^2 + 0.244 \times 3.53^3\right) - 600 \times (3.53 - 1)$$

$$= -560.49 \text{ kNm}$$

Sagging bending moment at outer face of column A, $x = 0.85\,m$

$$= \frac{110}{6}\left(3.33 \times 0.85^2 + 0.224 \times 0.85^3\right) = \underline{46.85\ kNm}$$

Sagging B.M at outer face of column B, $x = 6.2\,m$

$$= \frac{110}{6}\left(3.33 \times 6.2^2 + 0.224 \times 6.2^3\right) - 600\,(6.2-1) - 900\,(6.2-6)$$

$$= \underline{\underline{112.88\ KNm}}$$

For point of Contraflexures, equate B.M to zero.

$$\frac{110}{6}\left(3.33x^2 + 0.244x^3\right) - 600\,(x-1) = 0$$

$$366.3x^2 + 26.84x^3 - 3600x + 3600 = 0$$

$$\underline{x = 5.71\,m \text{ and } 1.14\,m}$$

<u>Shear forces</u> :-

$$S.F = V_x = 110\left(1.11x + 0.122x^2\right)$$

shear force Just left to A, i.e $x = 1m$, $S.F = 110\,(1.11 + 0.122)$

$$= \underline{135.52\ KN}$$

S.F Just right to A, $= 110\,(1.11 + 0.122) - 600 = \underline{\underline{-464.48\ KN}}$

S.F Just right to B i.e $x = 6m$

$$= 110\left(1.11 \times 6 + 0.122 \times 6^2\right) - 900 - 600 = \underline{\underline{-284.28\ KN}}$$

S.F Just left to B $= 110\left(1.11 \times 6 + 0.122 \times 6^2\right) - 600 = \underline{\underline{615.72\ KN}}$

S.F at contraflexures, $x = 5.71m$ $= 110\left(1.11 \times 5.71 + 0.122 \times 5.71^2\right) - 600$

$$= \underline{534\ KN}$$

S.F at $x = 1.14m$ $= 110\left(1.11 \times 1.14 + 0.122 \times 1.14^2\right) - 600$

$$= \underline{\underline{-443.36\ KN.}}$$

**Step-3:- Effective depth of footing from bending compression:-**

Effective depth, $d = \sqrt{\dfrac{M_u}{0.138\, f_{ck}\, B_x}}$

$B_x = \cancel{B_1 + B_2 x}\; 1.11 + 0.244x$

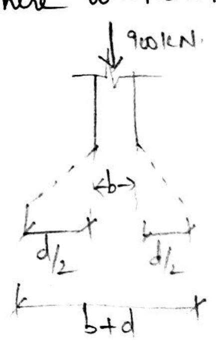
The Maximum bending moment occurs at a distance $x = 3.53\,m$

$B_x = 1.11 + 0.244 \times 3.53 = 1.97\,m$

$d = \sqrt{\dfrac{1.5 \times 560.49 \times 10^6}{0.138 \times 20 \times 1970}} = 393\,mm \simeq 400\,mm$

**Step-4:- Depth from punching shear (or) Two way shear consideration:-**

Punching shear will be maximum near Column B. where $w = 1.5 \times 900$ KN




Side $b_o = b + d = 400 + d$

Punching shear, $F_u = 1.5\,(w_2 - p_o b_o^2)$

$= 1.5\left(900 - 110\left[(400+d)\times10^{-3}\right]^2\right)$

$= 1350 - 165\times10^{-6}\,(400+d)^2$ KN

$= 1.35\times10^{6} - 0.165\,(400+d)^2$ N

Nominal shear stress, $\tau_v = \dfrac{V_u}{4 b_o d}$

$= \dfrac{1.35\times10^6 - 0.165\,(400+d)^2}{4\,(400+d)\,d}$

Permissible shear stress $= K_s \tau_c = 1 \times 0.25\sqrt{f_{ck}} = 1.118\ N/mm^2$

$\therefore \dfrac{1.35\times10^6 - 0.165\,(400+d)^2}{4\,(400+d)\,d} = 1.118$

$1.35\times10^6 - 26400 - 0.165 d^2 - 132 d = 1788.8d + 4.472 d^2$

$\Rightarrow 4.637 d^2 + 1920.8\, d - 1323600 = 0$

$\Rightarrow d = 365.89\ mm$

$\therefore$ d = 400mm . Using 60mm effective Cover, D = 400+60 = 460mm

Keep D = 500mm, Effective depth d = 500 − 60 = 440mm

**Step - 5:- Reinforcement in longitudinal direction:-**

(a) Reinforcement for hogging bending moment :-

$$M_u = 0.87\ f_y\ A_{st}\ d\left(1 - \frac{f_y\ A_{st}}{f_{ck}\ B_x d}\right)$$

$$1.5 \times 560.49 \times 10^6 = 0.87 \times 415 \times A_{st} \times 440\left(1 - \frac{415\ A_{st}}{20 \times 1970 \times 440}\right)$$

$$= 158862\ A_{st} - 3.80\ A_{st}^2$$

$$A_{st} = 6216.67\ mm^2$$

Using 20mm dia bars, No. of bars = $\frac{6216.67}{\pi/4 \times 20^2} = 19.78 \simeq 20$ bars

These bars should be checked for development length at the point of Contraflexure to satisfy the criteria $\frac{M_1}{V_u} + L_0 \geq L_d$

$$A_{st\ prov} = 20 \times \pi/4 \times 20^2 = 6284\ mm^2$$

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ B_x} = \frac{0.87 \times 415 \times 6284}{0.36 \times 20 \times 1970} = 159.95\ mm$$

$$M_1 = 0.87\ f_y\ A_{st}\ (d - 0.42\ x_u)$$

$$= 0.87 \times 415 \times 6284\ (440 - 0.42 \times 159.95) = 849.5\ KNm$$

At the point of Contraflexure near Column B

Shear force $V_u = 1.5 \times 534 \times 10^3$

$L_0 = 12\phi$ (or) d which ever is greater

$= 12 \times 20 = 240$ (or) 440mm $= 440$mm

$L_d = 47 \times \phi = 940$mm

$$\frac{M_1}{V_u} + L_0 = \frac{849.5 \times 10^6}{1.5 \times 534 \times 10^3} + 440 = 1500mm > 940 mm.$$

All 20 bars are taken beyond the point of contra flexure for a distance of 440mm. out of these 12 bars are curtailed and remaing 8 bars are taken upto the either edge to serve as anchorage bars for the stirrups.

(ii) Reinforcement for Sagging bending moment :-

$$M_{2u} = 1.5 \times 112.88 \times 10^6 = 169.32 \times 10^6 \text{ N mm}$$

width of footing near outer face of column B = $B_x = 1.118 + 0.244 \times 6.2$

$= 2.63 m$

$$169.32 \times 10^6 = 158862 A_{st} - 2.848 A_{st}^2$$

$$A_{st} = 1087. mm^2$$

Minimum reinforcement $= 0.12\%$ BD $= \frac{0.12}{100} \times 2630 \times 500$

$= 1578 \ mm^2$

Using 12mm dia bars, No. of bars $= \frac{1578}{\pi/4 \times 12^2} = 13.95 \simeq 14$ bars.

$$A_{st} \text{ provided} = 14 \times \pi/4 \times 12^2 = 1583.56 \ mm^2$$

$$x_u = \frac{0.87 \times 415 \times 1583.56}{0.36 \times 20 \times 2630} = 30.19 \ mm$$

$$M_2 = 0.87 \times 415 \times 1583.56 (440 - 0.42 \times 30.19) = 244.31 \ KNm$$

$$V_u = 1.5 \times 534 = 801 \ KN$$

$$L_0 = 12 \times 12 \ (or) \ 440 = 440 mm$$

$$L_d = 47 \times 12 = 564 mm$$

$$\frac{M_1}{V_u} + L_0 = \frac{244.31 \times 10^6}{801 \times 10^3} + 440 = 745 \ mm > 564 mm.$$

Hence safe.

(iii) Reinforcement for sagging B.M near outer face of Column A :-

$M_{3u} = 1.5 \times 46.85 \times 10^6 = 70.27 \times 10^6$ Nmm

width of footing near outer face of Column A, $B_x = 1.11 + 0.244 \times 0.85$

$B_x = 1.317 \simeq 1.32m$

$70.27 \times 10^6 = 1588\cancel{62}\, A_{st} - \overset{5.67}{\cancel{0.056}}\, A_{st}^2$

$A_{st} = 449.54$ $mm^2$

Minimum reinforcement $= 0.12\%\ B_x D = \frac{0.12}{100} \times 1320 \times 500$

$= 792$ $mm^2$

using 12mm dia bars, No. of bars $= \frac{792}{\pi/4 \times 12^2} = 7$ bars.

$x_u = 30.08$ mm, $M_3 = 122.17$ KNm

$V_u = 801$ KN, $L_o = 440$mm, $L_d = 564$ mm

$\frac{M_1}{V_u} + L_o = \frac{\overset{122.17}{\cancel{70.27}} \times 10^6}{801 \times 10^3} + 440 = 5\overset{92}{\cancel{27}}$ mm $>$ 564 mm

Hence ~~not~~ safe.

~~Let us assume $\frac{x_u}{d} = 0.3$ for under reinforced section~~

~~$x_u = 0.3 \times 440 = 132$ mm~~

~~$M_3 = 0.87 \times 415 \times \pi/4 \times 12^2 \times n\ (440 - 0.42 \times 132)$~~

~~$= 15.7 \times 10^6\, n$ Nmm~~

~~$\frac{15.7 \times 10^6\, n}{801 \times 10^3} + 440 = 564 \Rightarrow n =$~~

step-6 :- check for one way shear :-

In the Cantilever portion, test for one way shear is made at a distance $d = 440$mm from the Column face

$\therefore$ Distance $x$ from outer face of Column B $= 6.2 + 0.44$

$= 6.64$m.

At this section, shear force is extremely small, Hence shear stress will be small. Similarly in the cantilever portion left to column, A.

In between A and B, crack can occur at the point of contraflexure, where shear force = 534 kN. and $x = 5.71\,m$

$$B_x = 1.11 + 0.244 \times 5.71 = 2.5\,m$$

Nominal shear stress, $\tau_v = \dfrac{V_u}{B_x d} = \dfrac{1.5 \times 534 \times 10^3}{2500 \times 440} = 0.728\ N/mm^2$

$$\frac{100\,A_{st}}{B_x d} = \frac{100 \times 20 \times \pi/4 \times 20^2}{2500 \times 440} = 0.57\,\%$$

14 12 ← check

Permissible shear stress $= K\,\tau_c = 1 \times 0.482 = 0.482\ N/mm^2$

$\tau_v > K\,\tau_c$

Hence shear reinforcement is required.

Using 8 mm diameter 8 legged stirrups,

spacing $S_v = \dfrac{0.87\,f_y\,A_{sv}\,d}{V_{us}}$

$$= \frac{0.87 \times 415 \times 8 \times \pi/4 \times 8^2 \times 440}{1.5 \times 534 \times 10^3} = 79.76\,mm$$

Provide 8 mm diameter 8 legged stirrups @ 70 mm c/c.

Step - 7 :- Transverse reinforcement :-

(i) For Column A :-

The footing will bent transversely near each column face.

$B_x$ is the width of footing at the centre of column A ; $x = 1\,m$

$B_x = 1.11 + 0.244 = 1.354$ m

Projection 'a' beyond column face of $A = \frac{1}{2}(1354 - 300) = 527$ mm

Width of bending strip $(B_1) = b + 2d = 300 + 2 \times 440 = 1180$ mm

Net upward pressure, $p_0' = \frac{W_1}{B_x \times B_1} = \frac{600 \times 10^3}{1354 \times 1180} = 0.375 \ N/mm^2$

$= 375.5 \ KN/m^2$

Maximum bending will occur at the face of the column

$M = p_0' \times a^2/2 = 375.5 \times \frac{0.527^2}{2} = 52.14$ KNm

$M_u = 1.5 \times 52.14 = 78.22$ KNm

$d = \sqrt{\frac{78.22 \times 10^6}{0.138 \times 20 \times 1000}} = 168.34 \text{ mm} < 440 \text{ mm}$

Hence the provided depth is sufficient.

The transverse reinforcement will be provided above the longitudinal reinforcement

$d = 440 - 12 = 428$ mm

$78.22 \times 10^6 = 0.87 \times 415 \ A_{st} \times 428 \left(1 - \frac{415 \ A_{st}}{20 \times 1000 \times 428}\right)$

$= 154529.4 \ A_{st} - 7.49 \ A_{st}^2$

$A_{st} = 519.25 \ mm^2$, $A_{st \, min} = 0.12/100 \times 1000 \times 500 = 600 \ mm^2$

Using 8 mm dia bars, spacing $= ~~217.8~~$ 96.8 mm

Provide ~~12~~ 8 mm dia bars @ 90 mm c/c. The reinforcement provided above should have sufficient development length, $L_d = 47 \times 8 = ~~564~~$ 376 mm

Using 60 mm clear cover on the sides, length of bar available

$= 527 - 60 = 467 \text{ mm} > 376 \text{ mm}.$

(ii) For Column B :-

$B_x$ at centre of column B, $x = 6m$

$B_x = 1.11 + 0.244 \times 6 = 2.574$ m

Projection $a = \frac{1}{2}(2574 - 400) = 1087$ mm

width of bending strip $B_1 = b + 2d = 400 + 2 \times 440$

$= 1280$ mm

Net upward Pressure, $p_o' = \frac{W_2}{B_x \times B_1} = \frac{900 \times 10^3}{2574 \times 1280} = 273.16$ kN/m$^2$

$M_u = 1.5\, p_o'\, a^2/2 = 1.5 \times 273.16 \times \frac{1087^2}{2} = 242$ kNm

$d_{required} = \sqrt{\frac{242 \times 10^6}{0.138 \times 20 \times 1000}} = 296.15$ mm $< 440$mm.

Hence provided depth is sufficient.

$d = 440 - 12 = 428$mm for transverse reinforcement

$242 \times 10^6 = 154529.4\, A_{st} - 7.49\, A_{st}^2$

$A_{st} = 1707.33$ mm$^2$

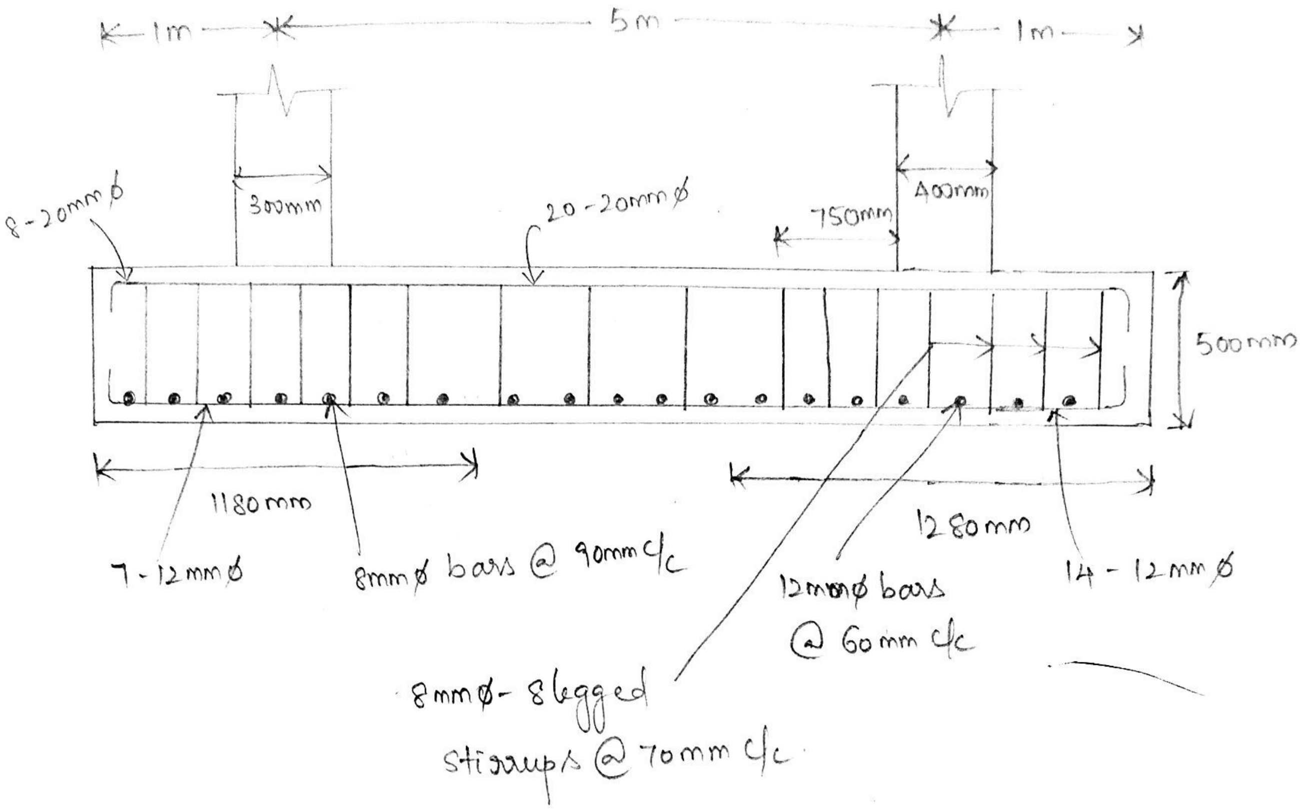
Using 12mm dia bars, spacing $= 66.25$ mm

Provide 12mm dia bars @ 60mm c/c.

$L_d$ required $= 47 \times 12 = 564$ mm

$L_d$ available $= 1087 - 60 = 1027$mm $> 564$ mm

Hence safe.




Reinforcement Details